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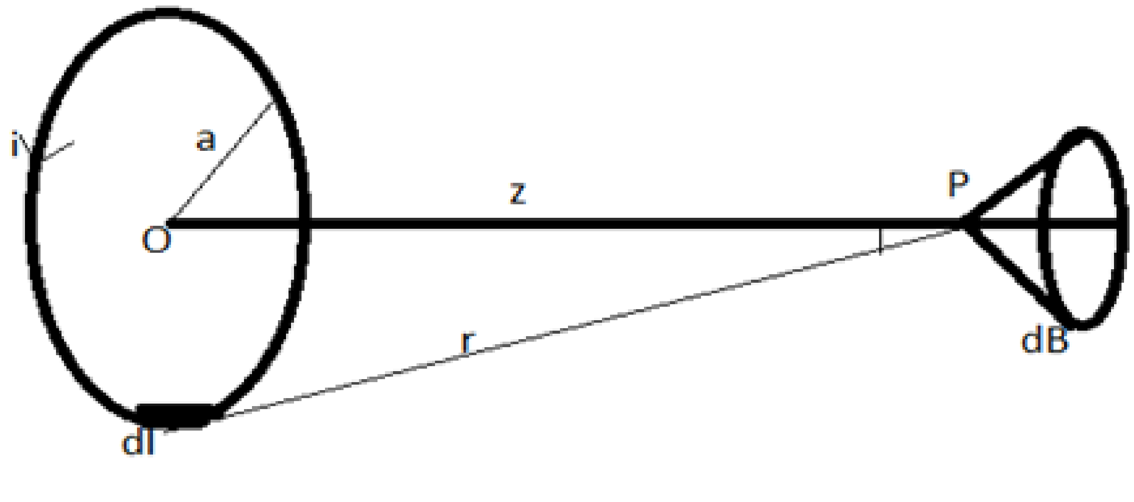

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In this Issue

Why nature made chemical potential of photons zero, for a black body?

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Abstract

Chemical potential of photons in a black body is zero. Thermodynamic interpretations for this can be seen in books and publications[1, 2, 3]. In this short communication we are trying to find out the reasons of making zero chemical potential of photons in a hot body from the quantum statistical mechanics view point.

1 Introduction

In thermodynamics, chemical potential forms a central concept and has marked its name in almost all the branches of science and technology. The knowledge of chemical potential of a material helps to obtain almost all its thermodynamic properties at a given temperature and pressure. Every substance has a tendency to change, may be a chemical reaction like rusting of iron or a phase change like evaporation of water, it is the chemical potential which controls them. Josiah Willard Gibbs[4] formally introduced the concept of chemical potential in his paper "On the Equilibrium of Heterogeneous Substances." Initially Gibbs termed it as 'intrinsic potential' but later it was Wilder Dwight Bancroft who coined it as 'chemical potential'[5]. Gibbs introduction of chemical potential marked the birth of chemical thermodynamics and made it possible to apply thermodynamics to material science and engineering. Maxwell[6] identified temperature, pressure, and chemical potential as potentials more than 140 years ago.

2 Classical and quantum systems

There are two types of physical phenomena which govern our lives, one is called classical phenomenon and the other is called quantum phenomenon. Classical phenomena are the physical phenomena which is natural to a human being and we experience its effects directly. Newton's laws are classical. Quantum phenomena are phenomena which occur at the atomic level or at the microscopic level and are indirectly perceived. Black body radiation, photoelectric effect etc are quantum phenomena. Laws of micro world control these effects. Historically we have classical statistical mechanics first and to get the quantum properties we made modifications in the classical statistical mechanics by incorporating the following nature of particles.

1. Wave particle duality
2. Spin
3. Identical nature of micro particles

For a high temperature system or for a classical system

$$\frac{N}{V}\lambda^3 \ll 1$$

where λ is the de Broglie thermal wavelength and $\frac{N}{V}$ is the number density. If any system is not obeying this condition, it will be a quantum system. So

$$\frac{N}{V}\lambda^3 \gg 1$$

$$\frac{N}{V}\lambda^3 = 1$$

and

$$\frac{N}{V}\lambda^3 \approx 1$$

are some conditions for quantum systems. For non relativistic particles, the de Broglie wavelength[7]

$$\lambda = \frac{h}{(2\pi mkT)^{1/2}}$$

where m is the mass of the particle, T is the absolute temperature, k is the Boltzmann constant and h is the Planck constant. For electron, mass $m = 9.1 \times 10^{-31}$ kg and then

$$\lambda = 7.466 \times 10^{-8} \times \frac{1}{T^{1/2}}$$

3 Examples

Let us study some systems at different temperatures, the corresponding de Broglie thermal wavelength λ and also find $\frac{N}{V}\lambda^3$.

3.1 Electrons in semiconductors

For electrons in semiconductors, the number density can be approximately taken as [8]

$$\frac{N}{V} = 10^{25}/m^3$$

Then for different temperatures we get

Table I

T(K)	$\lambda^3(m^3)$	$\frac{N}{V}\lambda^3$	Classical/ Quantum
10	1.314×10^{-23}	1.314×10^{2}	Quantum
10^2	4.15×10^{-25}	4.15	Quantum
10^3	1.314×10^{-26}	1.314×10^{-1}	Quantum
10^4	4.15×10^{-28}	4.15×10^{-3}	Classical
10^5	1.314×10^{-29}	1.314×10^{-4}	Classical
10^6	4.15×10^{-31}	4.15×10^{-6}	Classical
10^7	1.314×10^{-32}	1.314×10^{-7}	Classical

From Table I we can see that at high temperatures electrons in semiconductors show classical nature.

3.2 Electrons in a metal

The number density of electrons is approximately taken as [9]

$$\frac{N}{V} = 10^{28}/m^3$$

Then we get

Table II

T(K)	$\lambda^3(m^3)$	$\frac{N}{V}\lambda^3$	Classical/ Quantum
10	1.314×10^{-23}	1.314×10^{5}	Quantum
10^2	4.15×10^{-25}	4.15×10^{3}	Quantum
10^3	1.314×10^{-26}	1.314×10^{2}	Quantum
10^4	4.15×10^{-28}	4.15	Quantum
10^5	1.314×10^{-29}	1.314×10^{-1}	Quantum
10^6	4.15×10^{-31}	4.15×10^{-3}	Classical
10^7	1.314×10^{-32}	1.314×10^{-4}	Classical

From Table II we can see that for free electrons at very high temperatures, the behavior is classical.

4 Black body radiations

All objects with a temperature above absolute zero emit energy in the form of electromagnetic radiations. A black body is a body which absorbs all radiations falling on it and when heated it emits all kinds of radiations. The intensity distribution of the thermal energy radiated by a black body depends on its temperature and wavelength or frequency and is given by Planck's distribution law [9]

$$u(\nu)d\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}d\nu$$

where $u(\nu)d\nu$ is the energy density between ν an $\nu + d\nu$ with ν is the frequency and c is the velocity of light. To get this expression for photons we took the chemical potential as zero.

5 Thermodynamic interpretations for $\mu = 0$

Many authors give different reasons for chemical potential to be zero. According to Pathria[1] "This is due to the fact that the total number of particles in the present case is indefinite. For then, their equilibrium number N has to be determined by the condition that the free energy of the system is at its minimum, that is, $\left(\frac{\partial A}{\partial N}\right)_{V,T} = 0$, which, by definition, implies that $\mu = 0$ and hence $z = 1$". K Huang[2] says "One would say that chemical potential is zero, because a photon can disappear into vacuum". R. E Kelly[3] says " The chemical potential of photon gas in equilibrium in a volume, V and at temperature T, is formally given by $\mu = 0$. The physical reason for setting chemical potential zero is that number of photons in the volume cannot be arbitrary. That is number of photons are constantly adjusted so that the photon gas is in thermal equilibrium with the constant temperature walls of the container. Even a gas of photon far out in space not contain a fixed number of photons, since photons can be annihilated or created in collision. Therefore, when writing the thermodynamic identity for photons gas the term μdN, in the equation of change in internal energy $dU = TdS - pdV + \mu dN$, μdN should be omitted since N cannot be held fixed anyway. This is formally consistent with the setting $\mu = 0$. ”

6 Why nature made $\mu = 0$ for photons ?

Let us look at the same issue in a different perspective. For photons the number density [8]

$$\frac{N}{V} = \int_0^\infty n(\nu)d\nu$$

Therefore,

$$\frac{N}{V} = \int_0^\infty \frac{u(\nu)d\nu}{h\nu}$$

where $u(\nu)d\nu$ is the energy density between ν and $\nu + d\nu$. Substituting Planck's distribution law we get

$$\frac{N}{V} = \frac{8\pi\nu^2}{c^3} \int_0^\infty \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1} d\nu$$

Putting $\frac{h\nu}{kT} = x$ and carrying out suitable substitution

$$\frac{N}{V} = 8\pi \left(\frac{kT}{hc}\right)^3 \int_0^\infty \frac{x^2}{e^x - 1} dx$$

$$\frac{N}{V} = 8\pi \left(\frac{k}{hc}\right)^3 \times 2.404 \;\; T^3$$

For photons the de Broglie wavelength,

$$\lambda = \frac{hc}{2\pi^{1/3}kT}$$

Then

$$\lambda = 4.92 \times 10^{-3} \times \frac{1}{T}$$

For photons at different temperatures

Table IV

T(K)	$\lambda^3(m^3)$	$\frac{N}{V}(/m^3)$	$\frac{N}{V}\lambda^3$
10	1.19×10^{-10}	2.016×10^{10}	2.399
10^2	1.19×10^{-13}	2.016×10^{13}	2.399
10^3	1.19×10^{-16}	2.016×10^{16}	2.399
10^4	1.19×10^{-19}	2.016×10^{19}	2.399
10^5	1.19×10^{-22}	2.016×10^{22}	2.399
10^6	1.19×10^{-25}	2.016×10^{25}	2.399
10^7	1.19×10^{-28}	2.016×10^{28}	2.399

From Table IV we see that $\frac{N}{V}\lambda^3$ is always a constant, which means that the photons are always quantum from very low temperatures to high temperatures.

7 Conclusions

Making chemical potential zero helps a black body to be a quantum system at all temperatures. In many systems number density will be fixed and it is the temperature that decides when the system has to be a quantum system, whereas photon producing systems are always quantum systems and this is possible only when chemical potential is set to zero. In short nature makes a photon a quantum particle by making its chemical potential zero.

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A Note on Teaching Complex Numbers

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Abstract

Complex numbers can be introduced in a more intuitive and accessible manner using geometric ideas. In this note, we present such an approach wherein the process of rotation on a plane, well known to students, is used to motivate the multiplication rule for complex numbers.

Complex analysis is an important part of the mathematical training for science and engineering students. However, for many beginning student in these fields, the algebra of complex numbers, in particular the multiplication rule, appears strange. They are shown a mysterious symbol i, and they are told that it is subject conveniently to the rule $i^2 = -1$. They are then told that complex numbers are defined as expressions of the form $x + iy$, where x and y are arbitrary real numbers. Next, the formula for multiplying two complex numbers is derived for them from the fact that i^2 is supposed to be -1. All this leaves much unease among such students. They have worked only with real numbers so far, so they expect to associate a physical quality (such as a length) with numbers. Pulling a mysterious symbol i out of thin air and announcing the convenient rule that it squares to -1 appears artificial and ungrounded to them.

It is possible to present complex numbers differently, in a manner that resonates with those science and engineering students who understand mathematics better through physical objects and physical processes. The existence of a number that squares to -1 falls out naturally in this approach! We describe that presentation in this note.

As a prelude, let us recall what multiplication on a real line involves. The tip of an arrow from the origin to a point on the real line represents the real number corresponding to that point. For instance, the number 1 requires an arrow of length 1 unit drawn from the origin along the positive real axis. The real number -2 requires an arrow of length 2 units drawn from the origin along the negative real axis. How do we interpret multiplication in this representation of real numbers by arrows? If we multiply a number k by the positive number 2, the length of the arrow representing k changes by a factor of 2. If we multiply by the negative number -1.5, the length becomes 1.5 times the original length and direction flips. Thus, heuristically, we can interpret multiplication as a process of scaling the first arrow by the length of the second arrow, and additionally flipping it if the second arrow points along the negative real axis. Of course, this process is symmetric: we can just as easily interpret multiplication as scaling the second arrow by the length of the first arrow and flipping it if the first arrow points along the negative real axis. The key though is that multiplication of real numbers can be modeled by the two physical processes of scaling and flipping of arrows.

Of course, the addition of two real numbers can be interpreted using arrows as well: to add 2 to 3 for instance, we simply translate the vector representing 2 so that its tail now lies at the tip of the vector representing 3. The tip of the resulting vector now rests at the number 5. The same procedure can be performed for adding, say, 2 to -3: we translate the vector representing 2 to the tip of the vector representing -3, and the tip of the resulting vector will now rest at the number -1.

To discuss complex numbers, we begin with a discussion on the xy plane familiar to everyone. Once again, a point on the plane can be represented by the tip of an arrow extending from the origin to that point on the plane. Like in the case of the real line, an arrow on the plane can be stretched or shortened, flipped to point in the opposite direction, and translated along a second arrow. But unlike in the case of the real line, an arrow on the plane *can also be rotated.* It is this last feature — rotation — that allows us to present complex multiplication via a purely physical phenomenon! Moreover, rotation will reveal very naturally that there is a physical object (in fact, two!) whose square turns out to be -1.

We now *define* complex numbers to simply be arrows, also called *vectors*, on the xy plane, with their tails at the origin. Thus, each complex number is represented by an ordered pair (x, y) representing the tip of the vector. We add two complex numbers $\overline{u}$ and $\overline{v}$ by simply translating the vector $\overline{v}$ along the vector $\overline{u}$ till its tail lies at the tip of $\overline{u}$. See Figure 1. The arrow drawn from the origin to the tip of this translated arrow will represent the addition $\overline{u} + \overline{v}$. This is entirely analogous to how we interpret addition of two real numbers when they are represented as arrows along the real axis. This process is also known as the parallelogram law for addition of vectors.

It is easy to see using elementary geometry that if $\overline{u} = (x, y)$ and $\overline{v} = (a, b)$ then $\overline{u} + \overline{v} = (x + a, y + b)$. This same formula, as also just the underlying process of translation, show that addition is commutative: $\overline{u} + \overline{v}$ is the same as $\overline{v} + \overline{u}$.

To introduce multiplication using this framework, let us begin with the alternative *polar form* of a complex number. Let

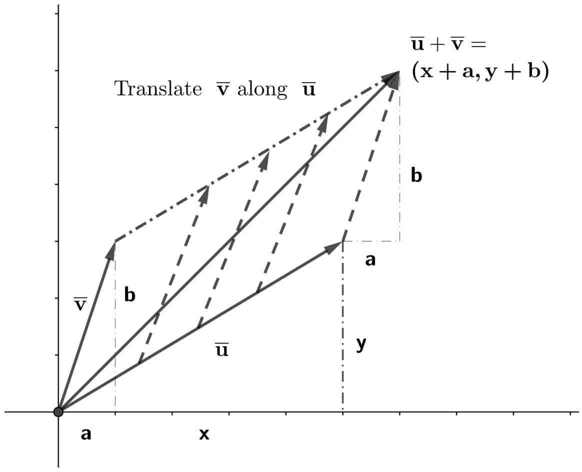

Figure 1: Parallelogram law for addition of vectors: $\overline{v}$ is translated along $\overline{u}$, parallel to itself, as shown by the dotted arrows.

$\overline{u} = (x, y)$ be a complex number, so $\overline{u}$ is an arrow with tail at the origin and tip at (x, y). Dropping a perpendicular from the tip to the x axis gives a right triangle. See Figure 2. If r is the length of the arrow, the coordinates (x, y) can be re-expressed as

$$(x, y) = (r\cos\theta, r\sin\theta), \tag{1}$$

where θ is the counter-clockwise angle between the arrow and the positive x-axis as shown in Figure 2. (We will informally refer to θ as the *angle of the vector* $\overline{u}$.) Also, $r = \sqrt{x^2+y^2}$, which is the result of applying the Pythagoras theorem to the right triangle shown in Figure 2. Now, consider rotating the arrow further in the clockwise direction by ϕ. The point corresponding to the tip of the arrow is changed to, say, (x', y'). This resultant point can also expressed in terms of r and, the angles θ and ϕ as $(r\cos(\theta+\phi), r\sin(\theta+\phi))$, since the new vector now lies at a counterclockwise angle of $\theta+\phi$ with respect to the positive x-axis. Expanding the cosines and sines gives

$$(x', y') = (r\cos\theta\cos\phi - r\sin\theta\sin\phi, r\sin\theta\cos\phi - r\cos\theta\sin\phi). \tag{2}$$

Using Eq.1, we rewrite the previous expression as

$$(x', y') = (x\cos\phi - y\sin\phi, x\sin\phi + y\cos\phi). \tag{3}$$

This relation connects (x, y), the coordinates prior to the rotation, to (x', y'), the coordinates after the rotation.

Now here is the leap of imagination:

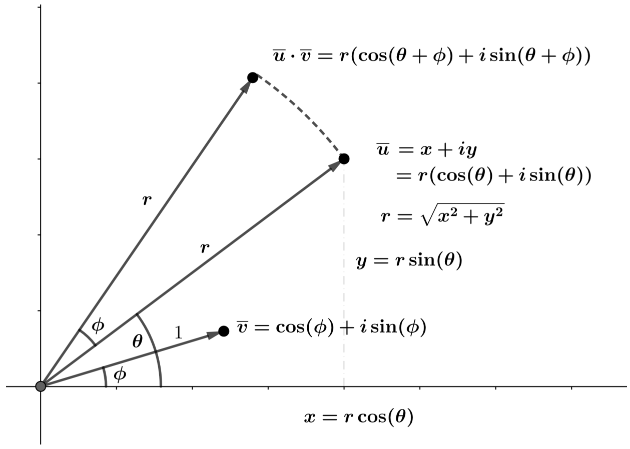

Figure 2: Description of rotation on a plane. The vector $\overline{u}$ is rotated by the angle ϕ of the vector $\overline{v}$.

Just as the physical processes of scaling and flipping of arrows on the real line represent multiplication of real numbers, could we not use the physical process of rotation of arrows on the plane to represent multiplication of complex numbers? In other words, can we interpret the rotation of $\overline{u}$ by the angle ϕ to be the result of performing a multiplication $\overline{u} \cdot \overline{v}$ for some as yet unknown complex number $\overline{v}$? If yes, what is a good candidate for $\overline{v}$? Since we are rotating $\overline{u}$ by an angle ϕ, and not changing the length in the process, a reasonable candidate for such a complex number would be $\overline{v} = (\cos\phi, \sin\phi)$. After all, this is the complex number represented by a vector at a counter-clockwise angle of ϕ from the positive x-axis, and of unit length.

Notice that if we were to write a for $\cos\phi$ and b for $\sin\phi$ (so $\overline{v} = (a, b)$), then $a^2 + b^2 = 1$ as $\overline{v}$ is of unit length. Also notice that in Equation 3, we can write $(x', y') = (xa - yb, xb + ya)$.

Motivated by this heuristic, we will now *decree* that for $\overline{u} = (x, y)$ and $\overline{v} = (a, b)$ where a and b satisfy $a^2 + b^2 = 1$ (so $a = \cos\phi$, $b = \sin\phi$, where $\phi = \cos^{-1}(a) = \sin^{-1}(b)$), the product $\overline{u} \cdot \overline{v}$ will be obtained as the result of rotation of $\overline{u}$ by the angle $\phi = \cos^{-1}(a) = \sin^{-1}(b)$, and will therefore have the formal expression $\overline{u} \cdot \overline{v} = (xa - yb, xb + ya)$.

What should we do when $\overline{v} = (a, b)$ and $a^2 + b^2 \neq 1$. Write $s = \sqrt{a^2 + b^2}$. If we assume $\overline{v} \neq (0, 0)$ then $s \neq 0$. So $\overline{v}$ is just the vector $\overline{v'} = (a/s, b/s)$, but scaled by s. The vector $\overline{v'}$ is of course of unit length, and as before is of the form $(\cos\phi, \sin\phi)$ where $\phi = \cos^{-1}(a/s) = \sin^{-1}(b/s)$. So, we now interpret multiplication of $\overline{u}$ by $\overline{v}$ as first multiplying $\overline{u}$ by $\overline{v'}$ (which is in effect rotation by ϕ) and then simply scaling the result by s. As we have seen, the result of multiplying $\overline{u} = (x, y)$ by $\overline{v'}$ (which is of unit length) is $(xa/s - yb/s, xb/s + ya/s)$, and the effect of scaling this by s of course yields back the previous expression $(xa - yb, xb + ya)$.

Finally, we will decree that multiplying $\overline{u}$ by the zero vector $(0,0)$ yields the zero vector $(0,0)$. Since $(0,0) = (x \cdot 0 - y \cdot 0, x \cdot 0 + y \cdot 0)$, we see that the formula for our multiplication of $\overline{u} = (x,y)$ by $\overline{v} = (a,b)$ in all cases reads $\overline{u} \cdot \overline{v} = (xa - yb, xb + ya)$.

Notice that our final product expression $(xa - yb, xb + ya)$ is unchanged if we swap x and y with a and b respectively. This shows that our multiplication operation is commutative: $\overline{u} \cdot \overline{v}$ is the same as $\overline{v} \cdot \overline{u}$.

This is a perfectly valid way to define the set of complex numbers! To summarize: We define them as vectors in the plane, with addition given by translating the first vector along the second vector, and multiplication given by rotation of the first vector by the angle of the second vector and scaling by the length of the second vector! Both addition and multiplication are commutative, as we have seen above. The real numbers show up in this representation as the vectors along the x-axis. Also, quite visibly, the formulas we get for the sum and product are exactly what we get when addition and multiplication are defined in the standard way: $(x + iy) + (a + ib) = (x + a) + i(y + b)$ and $(x + iy) \cdot (a + ib) = (xa - yb, xb + ya)$.

Notice now that we can use our physical representation to show that there is a complex number $\overline{u}$ whose square is -1. Note that -1 is the vector $(\cos \pi, \sin \pi)$ pointing along the negative x-axis of magnitude 1. We therefore need to search for a vector $\overline{u}$ of magnitude 1, so of the form $(\cos \phi, \sin \phi)$, which when rotated by its own angle ϕ (which corresponds to multiplying by itself) yields the vector $(\cos \pi, \sin \pi)$. Essentially, we need $\overline{u} \cdot \overline{u} = (\cos 2\phi, \sin 2\phi) = (\cos \pi, \sin \pi)$. Clearly $\overline{u} = (\cos \pi/2, \sin \pi/2) = (0,1)$ is a fine choice. So too is $\overline{u'} = (\cos 3\pi/2, \sin 3\pi/2) = (0,-1)$. We may pick $(0,1)$ as our primary choice for a square root of -1 and denote it as "i" (so the other choice is $-i$). This will hopefully take the mystery out of the complex number i by showing how naturally it arises as a physical object under these physical rules for multiplication!

One final fact about this representation: it is easy to use this representation to see how to divide $\overline{u}$ by a nonzero $\overline{v}$. Obviously, $\overline{u}/\overline{v} = \overline{u}\,(1/\overline{v})$, so it is sufficient to determine $1/\overline{v}$. When $\overline{v}$ is of unit length, it is clear what $(1/\overline{v})$ should be. If $\overline{v} = (\cos \phi, \sin \phi)$, then because $1 = (1,0) = (\cos 0, \sin 0)$, we need to rotate $\overline{v}$ *back* or *clockwise* by ϕ to bring it to angle 0. Alternatively, we need to rotate it counterclockwise by $-\phi$. The complex number $1/\overline{v}$ that will perform this rotation is $(\cos(-\phi), \sin(-\phi)) = (\cos \phi, -\sin \phi)$. This is of course just the usual "complex conjugate" of $\overline{v}$. When $\overline{v}$ is has magnitude $r \neq 1$, then we need to rotate by $-\phi$ and then further scale by $1/r$ to get a final length of 1, so $1/\overline{v} = (1/r)(\cos \phi, -\sin \phi)$.

It is also worth noting that the standard expression $x + iy$ for a complex number stems from picking $1 = (1,0)$ and $i = (0,1)$ as a basis for $\mathbb{R}^2$ and writing the vector with tip at (x, y) as $x \cdot 1 + y \cdot i = x + iy$!

We remark that the need for complex numbers in physics can be motivated *via* the Pauli algebra, which is the geometric algebra of the three dimensional Euclidean vector space, as well [1, 2]. Interestingly, Hamilton tried a similar approach, of defining multiplication on three-dimensional vectors by geometric processes, for constructing a three-dimensional number system in which one can meaningfully perform division. But he was not successful at creating one. However, he succeeded in formulating a *four*-dimensional number system in which division was possible. namely, the quaternions[3].

There are many excellent textbooks on complex analysis. We suggest the books by Flanigan[4] and Needham[5] for accessible and refreshing introductions to the subject.

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Voltmeters as EMF measurers?

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Abstract

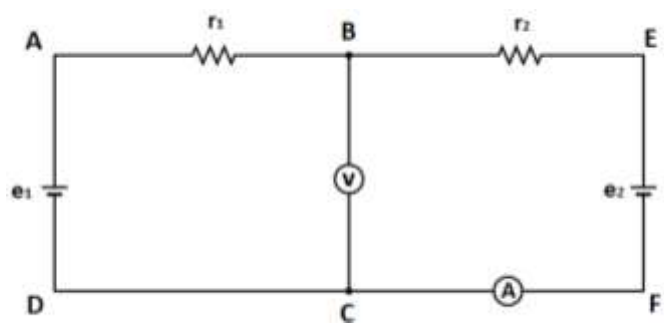
In high school physics it is usually taught that Voltmeters (DC) can only measure potential differences by drawing some current and (Circuit) Potentiometers can measure the EMF (Electromotive Force) of a cell. In this paper, the author discusses about a setup involving a voltmeter such that the voltmeter measures the EMF of the cell exactly.

1. Introduction

Electrical devices like voltmeters (DC) and potentiometers are generally introduced in high school physics as electrical equipment that measure potential differences in DC circuits. A voltmeter needs some power for itself to consume for measuring the emf of the cell and hence it will always indicate a voltage reading less than the actual EMF i.e., as the voltmeter itself draws some current so it cannot be used for measuring the EMF of the cell. Potentiometers get around with this by the *null method*. So, a potentiometer can give the exact value of potential difference across any two points in a circuit because it never draws any current from the circuit. Hence, potentiometers can measure the EMF of a cell exactly. In the following section, the author discusses about a possible set up where the voltmeter can measure the exact EMF of a cell by exploiting the *null method*.

2. Voltmeter as EMF-measurer

A voltmeter and a potentiometer do not have much difference between them because they both consist of resistances and galvanometers. The *apparent* difference that makes a potentiometer bit special from a voltmeter is that one can measure EMF of a cell exactly and this is solely possible as one implicitly applies the *null method* to measure potential differences in a potentiometer. No amount of current gets drawn into the measuring apparatus in *null method*, by definition. Consider the following circuit [1]:



In this circuit e_1 is a cell of (known) EMF e_1, e_2 is the cell whose EMF is to be determined (also $e_1 > e_2$), r_1 is a rheostat and r_2 is the total resistance of the circuit (wire) to which the cell of unknown EMF is connected. Except e_2 & r_2, the rest of the circuit is our *EMF measuring apparatus*, which, as evident from the above circuit diagram, contains a (DC) Voltmeter and an (DC) Ammeter (or Galvanometer would also work) besides e_1 & r_1. After connecting the circuit like the above circuit diagram, we adjust r_1 such that the ammeter shows a zero reading i.e., no current flows through the cell of EMF e_2. So, for this cell the lost volt is zero as well. Therefore, the voltmeter simply indicates the EMF of this cell i.e. the reading of the voltmeter is e_2, which is the exact value of the EMF of the cell whose EMF we wanted to determine in the first place.

3. Condition

In the previous section we concluded that if the ammeter shows a zero reading the voltmeter can easily measure the exact EMF of the cell. At this point the reader might have several questions in mind like – now that the voltmeter is measuring the exact EMF of the cell, is the voltmeter not drawing any current from the circuit? And the answer is both Yes and No. It is correct that still some amount of current (that basically passes through loop ABCDA) do flow through the voltmeter but on the other hand the voltmeter is not drawing any current from the circuit whose potential difference it is measuring (which is ensured by the zero reading of the ammeter). Henceforth, we find the sufficient and necessary condition such that the ammeter shows zero reading.

Note, as the voltmeter draws current from e_1, so by Ohm's law the current (I) passing through the voltmeter would be (as per the above circuit diagram):

$$\therefore I = \frac{e_1 - e_2}{r_1}, \qquad e_1 > e_2$$

This is the value for current passing through the voltmeter when the ammeter shows a zero reading. Now, apply Ohm's law to the segment BC to get the expression of current flowing through the voltmeter:

$$\therefore I = \frac{e_2}{R}$$

Where, R is the internal resistance of the voltmeter. Equating both the expressions we got for I, we get the following expression for r_1:

$$\therefore r_1 = R\left(\frac{e_1 - e_2}{e_2}\right)$$

This is our required necessary condition as from the above expression one can easily conclude that r_1 (which being a rheostat, we can adjust its value to make the reading of the ammeter equal to zero) is a function of e_2 i.e. EMF of the cell, whose EMF is to be measured and moreover, if by adjusting our rheostat we could obtain this value of the resistance (RHS) such that the null condition prevails, then this will serve as a sufficient condition for our problem in hand. Note that the current I is not in our control and we do not need to know e_1 or R to achieve this condition.

4. Remarks

From the analysis in the previous section we can draw some conclusions as follows –

- None of the parameters (e_1, r_1, R) of our *EMF measuring apparatus* circuit depend on r_2 in any way whatsoever.
- $e_1 > e_2$ i.e., the EMF of the cell in our *EMF measuring apparatus* must be greater than the EMF of the cell, whose EMF is to be determined. This result is quite intuitive because e_1 kind of provides some current that cancels the current coming from e_2 from the opposite direction and at the same time feeds the voltmeter with necessary power (by letting it to draw current) for its operation.
- From the expression of r_1 it is also clear that r_1 can be less than $(e_1 < 2e_2)$, greater than $(e_1 > 2e_2)$ or even equal $(e_1 = 2e_2)$ to R. That is why in our apparatus it was suggested to use a rheostat rather than a fixed-value resistor.
- The readings of our *EMF measuring apparatus* and that of a voltmeter (identical to the one used in the former or may be using the same one) can be compared to determine the amount of current drawn by the voltmeter and the percent error of the voltmeter in determining the exact EMF of a cell can be evaluated as well.
- Perhaps, the only limitation in the proposed method is the difficulty that varying rheostat is much less sensitive than adjusting the jockey on a long wire of a potentiometer.

5. Conclusion

Therefore, through the above analysis, we have successfully worked out the conditions and the apparatus design in which a voltmeter also can exactly measure the EMF of a cell. The apparatus design and proposed method to measure the EMF of a cell with a voltmeter should be achievable quite easily in a high school laboratory setting and it avoids the long potentiometer setup as well. Moreover, by understanding the above circuit design and exercise the author wants to impress upon the significance of the *null method* that is implicitly instrumental in measuring the EMF of the cell in both the cases of our *EMF measuring apparatus* and the standard circuit potentiometer apparatus. In our *EMF measuring apparatus* as well the *null method* is applied when the ammeter reading is set equal to zero.

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Innovative Approach to Determine the Radius of Circular Coil Using Cellphone Sensor

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Abstract

Magnetic field is induced in a current carrying coil, which can be measured by deflection θ (theta) of a magnetic needle in a compass. Conventional method to determine the radius of circular coil is by using compass. An innovative approach to find the variation of magnetic field intensity with distance and hence radius of circular coil was applied using cellophane sensor. In present work, a cellphone sensor has been used to determine the magnitude of magnetic field induced in the coil which is proportional to the current passing in the coil. Radius of circular coil was determined by the Gaussian nature of magnetic field versus distance curve. Variation in maximum magnetic field with current and the results for radius of the circular coil are reported.

Keywords: Cellphone sensor, Magnetic field, Circular coil

1. Introduction

Almost all cellphones include sensors for motion, light, sound, magnetic field and ambient pressure. There is an accelerometer, gyroscope, magnetometer, proximity sensor, light sensor, touchscreen sensor etc.in a cellphone [1-3]. A magnetometer measures the magnetic field in three dimensions and can also be used to measure the total magnetic field. The magnetometer sensor uses the solid-state technology and has a small Hall-effect sensor that detects the Earth's magnetic field along three axes X, Y and Z. The Hall-effect sensor produces voltage in response to a magnetic field which is proportional to the strength and polarity of the magnetic field along the axis each sensor is directed. The voltage produced is then converted to digital signal representing the magnetic field intensity. Other techniques used for magnetometer includes magneto resistive devices which change the measured resistance based on changes in the magnetic field. Magnetometer is also able to detect the position of cellphone, tilt and movement with the help of solid-state sensors in the cellphone. It is enclosed in a small electronic chip. In 1831, Michael Faraday reported on a series of experiments, including three that can be characterised as (a) He pulled a loop of wire to the right through a magnetic field, a current flowed in the loop. (b) He moved the magnet to the left, holding the loop still. Again, a current flowed in the loop and (c) With both the loop and the magnet at rest, he changed the strength of the field. Once again, current flowed in the loop [4-6].

An important current configuration in the study of magnetostatics is a circular loop carrying a current (i). it can be readily produced in the laboratory because it a closed path along which a steady-current flow continuously. The single loop idea can be extended to a solenoid which is nothing but a succession of single loops on the same axis. The Biot-Savart law and the superposition principle can be used to calculate the magnetic induction of a loop along its axis. Figure 1 shows a point P at a distance z from the centre O of a circular coil of radius a. OP is the axis of the coil and the current is anticlockwise as seen from P. We consider an element dl of the circle and calculate dB at P due to idl.

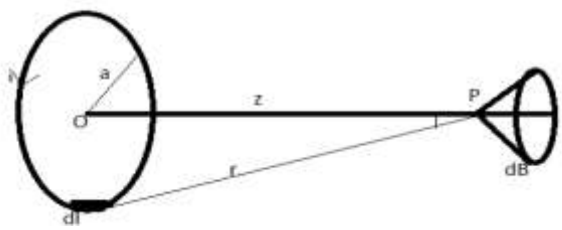

FIG. 1: Magnetic induction along the axis of a circular current loop of radius "a".

dB will be perpendicular to the plane containing dl and r. This is a plane passing through P and tangent to the circle at dl, the angle between dl and r is 90°. The vector dB which is normal to the plane of dl and r is also shown. Clearly if this makes an angle θ with z-axis, dB makes an angle (90- θ) with the z-axis.

The Biot-Savart law gives,

$$dB = \frac{\mu_0}{4\pi}\frac{id\boldsymbol{l} \times \boldsymbol{r}}{r^3} \quad \text{....1.1}$$

$$= \frac{\mu_0}{4\pi}\frac{idl}{[z^2 + a^2]} \quad \text{....1.2}$$

The circular loop may consider to be made of equal elements dl. Each element produces a field dB of magnitude given by above equation. The direction of each vector lies on the curved surface of a cone as shown in the figure. Thus, summation of dB is the resultant of all these vectors dB oriented symmetrically about the z-axis. The direction of their resultant must be along the z-axis. Once the direction of resultant of several vectors are known, the magnitude of the resultant can be evaluated by adding up the components of those vectors along the direction of their resultant. Now, only one integral is required. In this case, we have only to evaluate summation of dB_z. The dB_z is obtained by multiplying the right hand side of the above equation by "sin θ =a/((a^2+z^2)$^{0.5}$)". Replace dl by the circumference of the loop 2πa. A convenient expression for B at any point on the axis is,

$$B = \frac{\mu_0 i a^2}{2L^3} \quad \text{....1.3}$$

Where, L is the distance between the field point and any point on the loop. This L is same as r. According to Faraday, when the circular coil is at rest, the variation in current passing through the coil causes a change in strength of induced magnetic field [5-8]. The magnetic field varies with distance as $B=\mu_0 IA/(2\pi x^3)$, where I, is the current, A is the area, B is the magnetic field along the axis of the coil and x is the distance from the coil. Here area is constant for a coil; thus, for a constant value of current, magnitude of magnetic field varies inversely as distance. Similarly, for a particular distance, magnetic field varies as current [4-5]. Maximum magnetic field is observed at x=0. This is observed in the graph which is of Gaussian nature [9-10].

2. Experimental Details

A coil whose number of turns can vary was used in the present experiment. A calibrated flat base was arranged perpendicular to the plane of coil and passing through the centre of coil. Cellphone was mounted on the moving bar horizontally such that the sensor of the cellphone was just below the coil if observed from the top. The cellphone was tied to the moving bar with the help of string. The calibrated base read "0" where the coil coincides the position of sensor. The number of turns of the coil were set to 550.The circuit was connected as per the diagram shown in Figure 2. The current in the ammeter was adjusted with the help of rheostat. Corresponding reading of voltage was noted from power supply. The cellphone mounted on moving bar was kept 20 cm away from the centre of coil along its axis. The readings of magnetic field for every 1.0 cm distance were taken by keeping the constant current. Then, the cellphone was moved away from the centre of coil in the same direction as before but on the other side. Physics Tool android application was used to measure the magnetic field.

Then the current was increased with the help of rheostat. Corresponding reading of voltage was noted again and the cellphone was moved over the calibrated base and the procedure was repeated for entire scale. Similarly, a set of readings for one more value of current was taken and recorded. Physics Tool android application has provision to save the readings in digital form i.e., an Excel sheet.

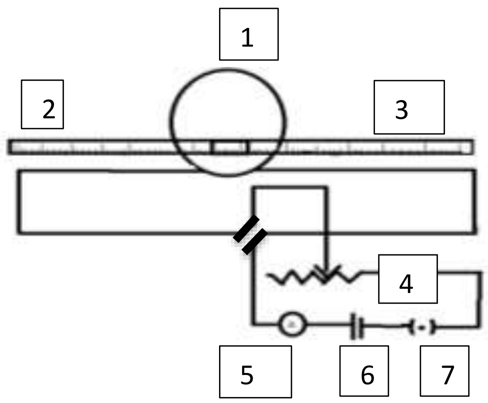

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FIG. 2: Experimental set up: (1) circular coil (2) calibrated horizontal base (3) cellphone-sensor (4) rheostat (5) ammeter (6) power supply and (7) key

3. Results and Discussion

3.1 *Magnetic Field with cellphone sensor*

The power supply was switched on. The current in the ammeter was adjusted with the help of rheostat and was kept 38mA. Corresponding reading of voltage was noted from power supply. The cell-phone mounted on moving bar was kept 20 cm away from the centre of coil along its axis. The readings of magnetic field for every 1cm distance were taken keeping the current constant. Thus, according to the formula value of induced magnetic field increased when the distance of cell-phone from the centre of coil was decreased. At the centre maximum magnetic field was observed as per Biot-Savart’s law i.e., B_0 was observed to be 1.49 Gauss. As the cell-phone was moved away from the centre of coil in the same direction as before, the value of induced magnetic field decreased. At 20 cm, from the centre of coil, the minimum value of induced magnetic field was observed. This gave rise to Gaussian nature of graph of magnetic field versus distance.

Thereafter, the current was increased to 60 mA with the help of rheostat. Corresponding reading of voltage i.e., 19.6 V was noted and the cellphone was moved over the calibrated base yielding one more curve of Gaussian nature (induced magnetic field versus distance). The maximum induced magnetic field i.e., B_0 was observed to be 2.23 Gauss. Further, current was again increased to 90 mA and corresponding voltage was noted as 28.9 V, the procedure was repeated with maximum value of induced magnetic field has been observed 3.22 Gauss. The comparative data are shown in Table 1.

Table 1: Radius of circular coil

Current (mA)	Voltage (V)	Max. Magnetic Field (B_0) GAUSS	$0.707 \times B_0$ GAUSS	RADIUS OF COIL (CMS)
38	12.3	1.49	1.07	10.4
60	19.6	2.23	1.57	10.2
90	28.9	3.22	2.27	10.1

Magnetic flux lines are induced due to current carrying loop. These flux lines are denser near the coil. The strength of magnetic field depends on the density of these flux lines. When the sensor (or cellphone) is 20 cm away from the coil, the density of these flux lines is less. Thus, low value of magnetic field intensity was observed. When the sensor approaches towards the coil, the magnetic flux lines become denser. Thus, greater value of magnetic field intensity was observed as we move closer to the coil carrying current. The flux lines are maximum at the centre of the coil. Therefore, the maximum value of magnetic field intensity was observed at the centre. As the sensor moves away from the coil in the other direction, the density of flux lines decreases. Thus, decreasing value of magnetic field intensity was observed as we move away from the current carrying coil. As the current passing through the coil is increased, a greater value of maximum magnetic field intensity was observed.

We have plotted a graph for variation of magnetic field with distance for three different values of currents and voltages. It was observed to be of Gaussian in nature. The maximum values of magnetic field intensities for three different values of current lies in the same YZ plane. The screenshot of sensor application with magnetic field and without field are shown in Figure 3 (a) and 3(b) respectively.

FIG. 3: Screenshot of sensor application (a) Without magnetic field (top) and (b) with magnetic field (down)

3.2 Determination of Radius of Circular Coil

Figure 4 shows the variation of magnetic field with distance from the centre of circular coil. To estimate the radius of circular coil, the 0.707 of maximum magnetic field (i.e., B_0) was taken and a horizontal line parallel to x-axis was drawn. Two perpendiculars were drawn from the points where the horizontal line intersected the graph. The distance between the feet of perpendiculars yielded radius of the circular coil.

Similarly, the values of radii for other two curves of Gaussian nature were obtained. Three values of radii for the corresponding values of current were obtained. Table 1 shows the comparative data for different values of currents i.e., radius of circular coil. It is found that the observed radii were very close to the actual radius of the circular coil which was 10 cms.

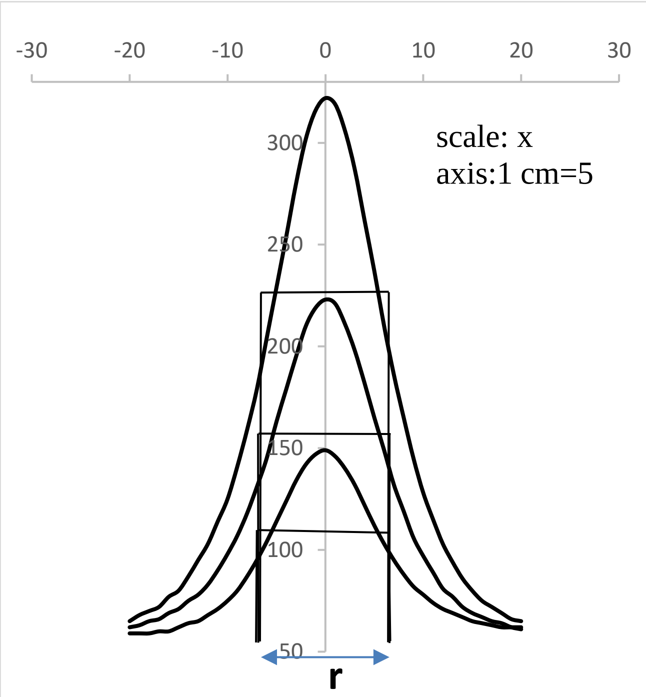

FIG. 4: Variation of magnetic field with distance using cellphone sensor

Conclusion

The present experiment is simple to understand the fundamentals of physics and can be carried out for an undergraduate laboratory. It is concluded from the above study that radius of the circular coil is found to be very close value of its true value. The expected error in measurement of magnetic field is less than 0.5μT. A cellphone sensor can be used to measure the magnetic field intensity with better resolution.

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