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EDITORIAL

As this issue of *Physics Education* reaches the readers, the Nobel prizes for the year 2018 had been announced. The physics prize is shared by Professors Arthur Ashkin, Gerard Mourou and Donna Strickland for their work in laser physics. This year's prize honours the scientists for their work on optical tweezers, which allows for optical tools to pick and manipulate atoms, small objects like bacteria and the like. It also honours the work on chirped pulse amplification that helps create highly intense laser radiation.

Apart from the science part of it, this years prizes have garnered attention for other reasons too. Professor Donna Strickland becomes the third woman to win the physics Nobel prize and the last time it happened was in the year 1963 when Maria Goeppert Mayer won the prize for her work on nuclear physics. Incidentally, adding to many first, Professor Ashkin became the oldest scientist at 96 years of age to have been awarded the Nobel prize.

Finally contributions of women to physics is beginning to recognised. This should inspire many more women to consider physics research and teaching as a career option. It is interesting to note that Maria Mayer did not have a salaried position until four years before she got the Nobel prize. Strickland's biographical entry in Wikipedia, the widely read open encyclopedia, was rejected because apparently she was not notable enough to deserve a place in Wikipedia. Clearly, it was and even now is not always a level playing for women physicists. Hopefully, this year's Nobel prize will bring about some long overdue changes for the women physicists.

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Physics Nobel Prizes at a Glance

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Abstract

The aim of this compilation is to present all the Physics Nobel prizes to the curious readers in a tabular form (easy to read and find) who are not only interested to know about the Nobel Laureates but also about the growth of physics concepts with time which have attracted the awards.

Keywords: Alfred Nobel, Nobel Prize, Physics

The Nobel Prize in Physics is a yearly award given by the Royal Swedish Academy of Sciences for those who conferred the most outstanding contributions for mankind in the field of physics [1]. It is one of the five Nobel Prizes established by the will of Alfred Nobel in 1895 and awarded since 1901; the others being the Nobel Prize in Chemistry, Nobel Prize in Literature, Nobel Peace Prize, and Nobel Prize in Physiology or Medicine. The first Nobel Prize in Physics was awarded to physicist Wilhelm Röntgen in recognition of the extraordinary services he has rendered by the discovery of the remarkable rays (or x-rays). This award is administered by the Nobel Foundation and widely regarded as the most prestigious award that a scientist can receive in physics. It is presented in Stockholm at an annual ceremony on 10 December, the anniversary of Nobel's death. Through 2017, a total of 111 Physics Nobel Prizes have been awarded to a total of 207 Physics Nobel Laureates (as John Bardeen has been awarded twice there are 206 individuals who have been awarded the Nobel Prize in Physics since 1901) and only two women have won the Nobel Prize in Physics: Marie Curie in 1903, and Maria Goeppert Mayer in 1963.

There is no other prize in the intellectual realm with the prestige of the Nobel Prizes. They also have a visibility that can hardly be compared to any other. In an age of science and technology in which we are gradually losing whole sets of values, fundamentally humanistic ones, the Nobel Prizes are one of our last bastions. We seek in them a reference, not only of excellence, but of honesty, enthusiasm, commitment to ideals, that inspires both laymen and professionals. Many of the winners were recognized among his peers as unique individuals, long before they were awarded the Prize. And afterwards they have continued to behave like people with great human qualities. That is why knowing about them (Nobel Laureates) and their contribution (awarded discoveries) towards science and technology can be a tremendous inspiration and example for the young students from all over the world. Of all the Nobel Prizes have a more fundamental character and there is a degree of truth associated with them and they all help to build what we might call the "Great Humanity". Their findings not only generate progress and allow society to develop, but also help us to know ourselves. Detailed knowledge about the laws and mechanisms governing Nature may have no immediate application, but make us aware of our own place in the Universe, help us to be more modest, more aware of our environment.

The aim of this compilation is to present all the Physics Nobel prizes to the curious readers in a tabular form (easy to read and find) who are not only interested to know about the Nobel Laureates but also about the growth of physics concepts with time which have attracted the awards.

Some most interesting facts about Nobel Prizes:

- In the statutes of the Nobel Foundation it says: "If none of the works under consideration is found to be of the importance indicated in the first paragraph, the prize money shall be reserved until the following year. If, even then, the prize cannot be awarded, the amount shall be added to the Foundation's restricted funds."
- Posthumous nominations can't be made for Nobel Prizes. If during consideration, the nominee dies, his name is removed. But if a person dies after being announced as the winner, a posthumous award is given.
- More than three people can't share a Nobel Prize.
- The Curies (Marie and Pierre Curie) comprised a very successful 'Nobel Prize family'. Marie Curie herself was awarded two Nobel Prizes - In 1903, she along with Pierre Curie (husband) was awarded half the Nobel Prize in Physics. In 1911 she was awarded the Nobel Prize in Chemistry. One of Marie and Pierre Curie's daughters, Irène Joliot-Curie, was awarded the Nobel Prize in Chemistry in 1935 together with her husband Frédéric Joliot.)
- To date, the youngest Nobel Laureate in Physics is Lawrence Bragg, who was 25 years old when he was awarded the Nobel Prize together with his father in 1915.
- The average age of Nobel laureates, across all prize categories, is 59. But the oldest prizewinner was 90-year-old Leonid Hurwicz, who won the Economics Nobel (technically called the Sveriges Riksbank Prize in Economics Sciences in Memory of Alfred Nobel) in 2007. The youngest winner is Malala Yousafzai. She won the Peace Prize in 2014 when she was 17 years old.
- There is often a substantial delay between when a scientist makes a Nobel-worthy discovery and receiving the award—the average time varies from 20 to 30 years, depending on the award category. Sometimes the wait is even longer
- Father & son awarded the Nobel Prize in Physics:
  William Bragg and Lawrence Bragg, 1915
  J. J. Thomson, 1906 and George Paget Thomson, 1937
  Niels Bohr, 1922 and Aage N. Bohr, 1975
  Manne Siegbahn, 1924 and Kai M. Siegbahn, 1981

Sr.No.	Year of Prize	Prize winners	Discovery
1.	2017	Rainer Weiss, Barry C. Barish and Kip S. Thorne	"for decisive contributions to the LIGO detector and the observation of gravitational waves"
2.	2016	David J. Thouless, F. Duncan M. Haldane and J. Michael Kosterlitz	"for theoretical discoveries of topological phase transitions and topological phases of matter"
3.	2015	Takaaki Kajita and Arthur B. McDonald	"for the discovery of neutrino oscillations, which shows that neutrinos have mass"
4.	2014	Isamu Akasaki, Hiroshi Amano and Shuji Nakamura	"for the invention of efficient blue light-emitting diodes which has enabled bright and energy-saving white light sources"
5.	2013	François Englert and Peter W. Higgs	"for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider"
6.	2012	Serge Haroche and David J. Wineland	"for ground-breaking experimental methods that enable measuring and manipulation of individual quantum systems"
7.	2011	Saul Perlmutter, Brian P. Schmidt and Adam G. Riess	"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"
8.	2010	Andre Geim and Konstantin Novoselov	"for groundbreaking experiments regarding the two-dimensional material graphene"
		Charles Kuen Kao	"for groundbreaking achievements concerning the transmission of light in fibers for optical communication"
		Willard S. Boyle and George E. Smith	"for the invention of an imaging semiconductor circuit - the CCD sensor"
		Yoichiro Nambu	"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"
		Makoto Kobayashi and Toshihide Maskawa	"for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"
11.	2007	Albert Fert and Peter Grünberg	"for the discovery of Giant Magnetoresistance"
12.	2006	John C. Mather and George F. Smoot	"for their discovery of the blackbody form and anisotropy of the cosmic microwave background radiation"
		Roy J. Glauber	"for his contribution to the quantum theory of optical coherence"
		John L. Hall and Theodor W. Hänsch	"for their contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique"
14.	2004	David J. Gross, H. David Politzer and Frank Wilczek	"for the discovery of asymptotic freedom in the theory of the strong interaction"
15.	2003	Alexei A. Abrikosov, Vitaly L. Ginzburg and Anthony J. Leggett	"for pioneering contributions to the theory of superconductors and superfluids"
		Raymond Davis Jr. and Masatoshi Koshiba	"for pioneering contributions to astrophysics, in particular for the detection of cosmic neutrinos"
		Riccardo Giacconi	"for pioneering contributions to astrophysics, which have led to the discovery of cosmic X-ray sources"
17.	2001	Eric A. Cornell, Wolfgang Ketterle and Carl E. Wieman	"for the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early

			fundamental studies of the properties of the condensates"
		Zhores I. Alferov and Herbert Kroemer	"for basic work on information and communication technology"
		Jack S. Kilby	"for his part in the invention of the integrated circuit"
19.	1999	Gerardus 't Hooft and Martinus J.G. Veltman	"for elucidating the quantum structure of electroweak interactions in physics"
20.	1998	Robert B. Laughlin, Horst L. Störmer and Daniel C. Tsui	"for their discovery of a new form of quantum fluid with fractionally charged excitations"
21.	1997	Steven Chu, Claude Cohen-Tannoudji and William D. Phillips	"for development of methods to cool and trap atoms with laser light"
22.	1996	David M. Lee, Douglas D. Osheroff and Robert C. Richardson	"for their discovery of superfluidity in helium-3"
		Martin L. Perl	"for the discovery of the tau lepton"
		Frederick Reines	"for the detection of the neutrino"
		Bertram N. Brockhouse	"for the development of neutron spectroscopy"
		Clifford G. Shull	"for the development of the neutron diffraction technique"
25.	1993	Russell A. Hulse and Joseph H. Taylor Jr.	"for the discovery of a new type of pulsar, a discovery that has opened up new possibilities for the study of gravitation"
26.	1992	Georges Charpak	"for his invention and development of particle detectors, in particular the multiwire proportional chamber"
27.	1991	Pierre-Gilles de Gennes	"for discovering that methods developed for studying order phenomena in simple systems can be generalized to more complex forms of matter, in particular to liquid crystals and polymers"
28.	1990	Jerome I. Friedman, Henry W. Kendall and Richard E. Taylor	"for their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics"
		Norman F. Ramsey	"for the invention of the separated oscillatory fields method and its use in the hydrogen maser and other atomic clocks"
		Hans G. Dehmelt and Wolfgang Paul	"for the development of the ion trap technique"
30.	1988	Leon M. Lederman, Melvin Schwartz and Jack Steinberger	"for the neutrino beam method and the demonstration of the doublet structure of the leptons through the discovery of the muon neutrino"
31.	1987	J. Georg Bednorz and K. Alexander Müller	"for their important break-through in the discovery of superconductivity in ceramic materials"
		Ernst Ruska	"for his fundamental work in electron optics, and for the design of the first electron microscope"
		Gerd Binnig and Heinrich Rohrer	"for their design of the scanning tunneling microscope"
33.	1985	Klaus von Klitzing	"for the discovery of the quantized Hall effect"
34.	1984	Carlo Rubbia and Simon van der Meer	"for their decisive contributions to the large project, which led to the discovery of the field particles W and Z, communicators of weak interaction"
		Subramanyan Chandrasekhar	"for his theoretical studies of the physical processes of importance to the structure and evolution of the stars"

		William Alfred Fowler	"for his theoretical and experimental studies of the nuclear reactions of importance in the formation of the chemical elements in the universe"
36.	1982	Kenneth G. Wilson	"for his theory for critical phenomena in connection with phase transitions"
		Nicolaas Bloembergen and Arthur Leonard Schawlow	"for their contribution to the development of laser spectroscopy"
		Kai M. Siegbahn	"for his contribution to the development of high-resolution electron spectroscopy"
38.	1980	James Watson Cronin and Val Logsdon Fitch	"for the discovery of violations of fundamental symmetry principles in the decay of neutral K-mesons"
39.	1979	Sheldon Lee Glashow, Abdus Salam and Steven Weinberg	"for their contributions to the theory of the unified weak and electromagnetic interaction between elementary particles, including, inter alia, the prediction of the weak neutral current"
		Pyotr Leonidovich Kapitsa	"for his basic inventions and discoveries in the area of low-temperature physics"
		Arno Allan Penzias and Robert Woodrow Wilson	"for their discovery of cosmic microwave background radiation"
41.	1977	Philip Warren Anderson, Sir Nevill Francis Mott and John Hasbrouck van Vleck	"for their fundamental theoretical investigations of the electronic structure of magnetic and disordered systems"
42.	1976	Burton Richter and Samuel Chao Chung Ting	"for their pioneering work in the discovery of a heavy elementary particle of a new kind"
43.	1975	Aage Niels Bohr, Ben Roy Mottelson and Leo James Rainwater	"for the discovery of the connection between collective motion and particle motion in atomic nuclei and the development of the theory of the structure of the atomic nucleus based on this connection"
44.	1974	Sir Martin Ryle and Antony Hewish	"for their pioneering research in radio astrophysics: Ryle for his observations and inventions, in particular of the aperture synthesis technique, and Hewish for his decisive role in the discovery of pulsars"
		Leo Esaki and Ivar Giaever	"for their experimental discoveries regarding tunneling phenomena in semiconductors and superconductors, respectively"
		Brian David Josephson	"for his theoretical predictions of the properties of a supercurrent through a tunnel barrier, in particular those phenomena which are generally known as the Josephson effects"
46.	1972	John Bardeen, Leon Neil Cooper and John Robert Schrieffer	"for their jointly developed theory of superconductivity, usually called the BCS-theory"
47.	1971	Dennis Gabor	"for his invention and development of the holographic method"
		Hannes Olof Gösta Alfvén	"for fundamental work and discoveries in magnetohydro-dynamics with fruitful applications in different parts of plasma physics"
		Louis Eugène Félix Néel	"for fundamental work and discoveries concerning antiferromagnetism and ferrimagnetism which have led to important applications in solid state physics"
49.	1969	Murray Gell-Mann	"for his contributions and discoveries concerning the classification of elementary particles and their

			interactions"
50.	1968	Luis Walter Alvarez	"for his decisive contributions to elementary particle physics, in particular the discovery of a large number of resonance states, made possible through his development of the technique of using hydrogen bubble chamber and data analysis"
51.	1967	Hans Albrecht Bethe	"for his contributions to the theory of nuclear reactions, especially his discoveries concerning the energy production in stars"
52.	1966	Alfred Kastler	"for the discovery and development of optical methods for studying Hertzian resonances in atoms"
53.	1965	Sin-Itiro Tomonaga, Julian Schwinger and Richard P. Feynman	"for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles"
54.	1964	Charles Hard Townes, Nicolay Gennadiyevich Basov and Aleksandr Mikhailovich Prokhorov	"for fundamental work in the field of quantum electronics, which has led to the construction of oscillators and amplifiers based on the maser-laser principle"
		Eugene Paul Wigner	"for his contributions to the theory of the atomic nucleus and the elementary particles, particularly through the discovery and application of fundamental symmetry principles"
		Maria Goeppert Mayer and J. Hans D. Jensen	"for their discoveries concerning nuclear shell structure"
56.	1962	Lev Davidovich Landau	"for his pioneering theories for condensed matter, especially liquid helium"
		Robert Hofstadter	"for his pioneering studies of electron scattering in atomic nuclei and for his thereby achieved discoveries concerning the structure of the nucleons"
		Rudolf Ludwig Mössbauer	"for his researches concerning the resonance absorption of gamma radiation and his discovery in this connection of the effect which bears his name"
58.	1960	Donald Arthur Glaser	"for the invention of the bubble chamber"
59.	1959	Emilio Gino Segrè and Owen Chamberlain	"for their discovery of the antiproton"
60.	1958	Pavel Alekseyevich Cherenkov, Il´ja Mikhailovich Frank and Igor Yevgenyevich Tamm	"for the discovery and the interpretation of the Cherenkov effect"
61.	1957	Chen Ning Yang and Tsung-Dao (T.D.) Lee	"for their penetrating investigation of the so-called parity laws which has led to important discoveries regarding the elementary particles"
62.	1956	William Bradford Shockley, John Bardeen and Walter Houser Brattain	"for their researches on semiconductors and their discovery of the transistor effect"
		Willis Eugene Lamb	"for his discoveries concerning the fine structure of the hydrogen spectrum"
		Polykarp Kusch	"for his precision determination of the magnetic moment of the electron"
		Max Born	"for his fundamental research in quantum mechanics, especially for his statistical interpretation of the wavefunction"
		Walther Bothe	"for the coincidence method and his discoveries made therewith"
65.	1953	Frits Zernike	"for his demonstration of the phase contrast method, especially for his invention of the phase contrast

			microscope"
66.	1952	Felix Bloch and Edward Mills Purcell	"for their development of new methods for nuclear magnetic precision measurements and discoveries in connection therewith"
67.	1951	Sir John Douglas Cockcroft and Ernest Thomas Sinton Walton	"for their pioneer work on the transmutation of atomic nuclei by artificially accelerated atomic particles"
68.	1950	Cecil Frank Powell	"for his development of the photographic method of studying nuclear processes and his discoveries regarding mesons made with this method"
69.	1949	Hideki Yukawa	"for his prediction of the existence of mesons on the basis of theoretical work on nuclear forces"
70.	1948	Patrick Maynard Stuart Blackett	"for his development of the Wilson cloud chamber method, and his discoveries therewith in the fields of nuclear physics and cosmic radiation"
71.	1947	Sir Edward Victor Appleton	"for his investigations of the physics of the upper atmosphere especially for the discovery of the so-called Appleton layer"
72.	1946	Percy Williams Bridgman	"for the invention of an apparatus to produce extremely high pressures, and for the discoveries he made therewith in the field of high pressure physics"
73.	1945	Wolfgang Pauli	"for the discovery of the Exclusion Principle, also called the Pauli Principle"
74.	1944	Isidor Isaac Rabi	"for his resonance method for recording the magnetic properties of atomic nuclei"
75.	1943	Otto Stern	"for his contribution to the development of the molecular ray method and his discovery of the magnetic moment of the proton"
76.	1939	Ernest Orlando Lawrence	"for the invention and development of the cyclotron and for results obtained with it, especially with regard to artificial radioactive elements"
77.	1938	Enrico Fermi	"for his demonstrations of the existence of new radioactive elements produced by neutron irradiation, and for his related discovery of nuclear reactions brought about by slow neutrons"
78.	1937	Clinton Joseph Davisson and George Paget Thomson	"for their experimental discovery of the diffraction of electrons by crystals"
		Victor Franz Hess	"for his discovery of cosmic radiation"
		Carl David Anderson	"for his discovery of the positron"
80.	1935	James Chadwick	"for the discovery of the neutron"
81.	1933	Erwin Schrödinger and Paul Adrien Maurice Dirac	"for the discovery of new productive forms of atomic theory"
82.	1932	Werner Karl Heisenberg	"for the creation of quantum mechanics, the application of which has, inter alia, led to the discovery of the allotropic forms of hydrogen"
83.	1930	Sir Chandrasekhara Venkata Raman	"for his work on the scattering of light and for the discovery of the effect named after him"
84.	1929	Prince Louis-Victor Pierre Raymond de Broglie	"for his discovery of the wave nature of electrons"
85.	1928	Owen Willans Richardson	"for his work on the thermionic phenomenon and especially for the discovery of the law named after him"
		Arthur Holly Compton	"for his discovery of the effect named after him"
		Charles Thomson Rees Wilson	"for his method of making the paths of electrically

			charged particles visible by condensation of vapour"
87.	1926	Jean Baptiste Perrin	"for his work on the discontinuous structure of matter, and especially for his discovery of sedimentation equilibrium"
88.	1925	James Franck and Gustav Ludwig Hertz	"for their discovery of the laws governing the impact of an electron upon an atom"
89.	1924	Karl Manne Georg Siegbahn	"for his discoveries and research in the field of X-ray spectroscopy"
90.	1923	Robert Andrews Millikan	"for his work on the elementary charge of electricity and on the photoelectric effect"
91.	1922	Niels Henrik David Bohr	"for his services in the investigation of the structure of atoms and of the radiation emanating from them"
92.	1921	Albert Einstein	"for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect"
93.	1920	Charles Edouard Guillaume	"in recognition of the service he has rendered to precision measurements in Physics by his discovery of anomalies in nickel steel alloys"
94.	1919	Johannes Stark	"for his discovery of the Doppler effect in canal rays and the splitting of spectral lines in electric fields"
95.	1918	Max Karl Ernst Ludwig Planck	"in recognition of the services he rendered to the advancement of Physics by his discovery of energy quanta"
96.	1917	Charles Glover Barkla	"for his discovery of the characteristic Röntgen radiation of the elements"
97.	1915	Sir William Henry Bragg and William Lawrence Bragg	"for their services in the analysis of crystal structure by means of X-rays"
98.	1914	Max von Laue	"for his discovery of the diffraction of X-rays by crystals"
99.	1913	Heike Kamerlingh Onnes	"for his investigations on the properties of matter at low temperatures which led, inter alia, to the production of liquid helium"
100.	1912	Nils Gustaf Dalén	"for his invention of automatic regulators for use in conjunction with gas accumulators for illuminating lighthouses and buoys"
101.	1911	Wilhelm Wien	"for his discoveries regarding the laws governing the radiation of heat"
102.	1910	Johannes Diderik van der Waals	"for his work on the equation of state for gases and liquids"
103.	1909	Guglielmo Marconi and Karl Ferdinand Braun	"in recognition of their contributions to the development of wireless telegraphy"
104.	1908	Gabriel Lippmann	"for his method of reproducing colours photographically based on the phenomenon of interference"
105.	1907	Albert Abraham Michelson	"for his optical precision instruments and the spectroscopic and metrological investigations carried out with their aid"
106.	1906	Joseph John Thomson	"in recognition of the great merits of his theoretical and experimental investigations on the conduction of electricity by gases"
107.	1905	Philipp Eduard Anton von Lenard	"for his work on cathode rays"
108.	1904	Lord Rayleigh (John William Strutt)	"for his investigations of the densities of the most important gases and for his discovery of argon in connection with these studies"

109.	1903	Antoine Henri Becquerel	"in recognition of the extraordinary services he has rendered by his discovery of spontaneous radioactivity"
		Pierre Curie and Marie Curie, née Sklodowska	"in recognition of the extraordinary services they have rendered by their joint researches on the radiation phenomena discovered by Professor Henri Becquerel"
110.	1902	Hendrik Antoon Lorentz and Pieter Zeeman	"in recognition of the extraordinary service they rendered by their researches into the influence of magnetism upon radiation phenomena"
111.	1901	Wilhelm Conrad Röntgen	"in recognition of the extraordinary services he has rendered by the discovery of the remarkable rays subsequently named after him"
No Nobel Prize was awarded in 1916, 1931, 1934, 1940, 1941 and 1942. In 1916, 1940-42 it is quite reasonable to assume that there was no awarding due to the world wars.			

[1]. https://www.nobelprize.org/nobel_prizes/physics/laureates/

Locating Planets in Sky Using Manual Calculations

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Abstract

In this article, we describe a very simple technique to locate naked-eye planets in the sky, to an accuracy of $\sim 1°$, up to say, 2050 AD. The procedure, comprising just three steps, involves very simple manual calculations for planetary orbits around the Sun; all one needs are the initial specifications of planetary positions for some standard epoch and the time periods of their revolutions. After applying a small correction for the orbital ellipticity, appearance of a planet relative to Sun's position in sky, as seen by an observer from Earth, is found using a scale and a protractor (found inside a school geometry box).

1 Introduction

Quite often, seeing a bright star-like object in the evening sky (or in the morning sky for the early birds!) many of us would have wondered whether it is a planet and if so, which one. To be able to actually locate a planet in the sky is something that could be thrilling to most of us and occasionally it provides us an opportunity to impress our friends and acquaintances. Although daily planetary positions could be obtained from the professional ephemeris [1] or simply from the internet, yet it is very instructive and much more satisfying to be able to calculate these ourselves, starting from, say, one of Kepler's famous laws, which states that planets go around Sun in elliptical paths.

The path that Earth takes in the sky is called the ecliptic. The familiar Zodiac constellations are just divisions of the ecliptic into twelve parts. Sun as well as planets, as seen from Earth, also appear to move along the ecliptic and pass through the Zodiac constellations. The angle along the ecliptic is called longitude (denoted by λ), measured eastwards, that is, anti-clockwise, from $0°$ to $360°$. Its origin, $\lambda = 0°$, is known as the First Point of Aries, the position Sun occupies on the ecliptic around 21st of March every year.

The first step in our exercise would be to calculate the longitude of a planet as well as that of Earth around Sun. We initially consider the planets to revolve around Sun in uniform circular motion. However, this will entail a correction since the actual orbits are elliptical where the angular speed is not uniform. In second step we employ a pre-computed table to find the necessary corrections. From another table we also determine distances of planets from Sun. In a third and final step, we plot the distances along the corrected longitudes of chosen planet as well as that of Earth centred around Sun on a graph sheet or a chart, using a scale and a protractor (found inside a school geometry box) and measure the angle between the planet and Sun as seen from Earth, which allows us to locate the planet in the sky.

2 Step 1: Longitude in a circular orbit

First we consider the planets to move around Sun in circular orbits with uniform angular speeds. We need longitudes of planets in their orbits around Sun on some initial date. Here we take 1st January, 2000, 00:00 UT as our initial date [2] for which we have listed the longitudes (λ_i) of the planets (Table A1). Also listed in Table A1 is the period T (days) of revolution of each planet [3]. From T we get the mean angular speed of the planet as $\omega_0 = 360/T$ ($^\circ$/day). We denote the Mean Longitude of the planets in the imaginary circular orbit for subsequent dates as λ_0.

As an example, we calculate the mean longitude λ_0 for Venus on 1st October 2018.

The initial longitude of Venus (on 01.01.2000), $\lambda_i = 181.2^\circ$.

The mean angular speed of Venus, $\omega_0 = 1.60213^\circ/\text{day}$

No. of days between 01.01.2000 and 01.10.2018, $N = 365 \times 18 + 273 + 5 = 6848$ (including 5 leap days).

The mean angle traversed duration this period, $\omega_0 N = 1.60213 \times 6848 = 10971.4^\circ$.

So, on 01.10.18 the mean longitude of Venus, $\lambda_0 = 181.2 + 10971.4 = 11152.6^\circ$.

After taking out 30 complete orbits in integer multiple of 360°, we get mean longitude of Venus, $\lambda_0 = 11152.6 - 360 \times 30 = 352.6^\circ$.

As another example we also calculate the mean longitude of Jupiter on 01.10.18, $\lambda_0 = 34.3 + 0.08309 \times 6848 = 603.3^\circ$. After taking out one complete cycle, the mean longitude of Jupiter on 01.10.18 is $\lambda_0 = 243.3^\circ$.

We also need to calculate the mean longitude of Earth on 01.10.18 as $\lambda_0 = 100 + 0.98561 \times 6848 = 6849.5^\circ$. Or the mean longitude of Earth on 01.10.18 is $\lambda_0 = 9.5^\circ$ (after taking out 19 complete orbits).

3 Step 2: Correcting for the elliptical motion

The orbits of planets around Sun are elliptical and as a result their angular speeds are not exactly uniform. The corrections in the longitudes of some of the planets due to this variation in their angular speeds could be substantial. To correct for the elliptical motion, here we employ Table A2, computed based on the formulation derived in [4]. Entries for each planet in Table A2 also account for the orientation of its elliptical orbit within the ecliptic plane, specified by the longitude of the perihelion, where perihelion is the point closest to the Sun on the elliptical orbit of the planet.

The correction from the Table A2 for Venus for $\lambda_0 = 352.6$ is $-0.3°$. Therefore corrected longitude is $\lambda = 352.6 - 0.3 = 352.3°$.

The correction for Jupiter for $\lambda_0 = 243.3°$ is $-3.8°$, with corrected longitude $\lambda = 243.3 - 3.8 = 239.5°$.

Similarly we calculate the corrected elliptical longitude for Earth as $\lambda = 7.8°$.

4 Step 3: Calculating the elongation from Sun

The difference between the geocentric position of a planet and Sun is called the elongation (ψ) of the planet and it tells us about planet's position in the sky relative to that of the Sun. As we want to find the sky position of a planet, as seen by an observer located on Earth, we need to find the position of Earth too in the ecliptic. Since Earth longitude changes by $\sim 1°$ per day, we need Earth position for the same date and time as that of the planet we are interested in.

We have already calculated the ecliptic longitudes of Venus, Jupiter and Earth. For each of these planets we also need to find radii r, the distance from Sun, which is listed against λ_0 in Table A3 in AU (Astronomical Unit - the mean distance between Earth and Sun). For our chosen date of 01/10/2018, r is 0.73 A.U. for Venus, 5.37 for Jupiter and 1.00 A.U. for Earth.

Although one could employ a calculator to compute geocentric longitude and the elongation of the planet [4], however, a manual geometric construction could be much more illuminating. All one needs is a scale and a protractor, usually found in a school geometry box.

Now we plot on a graph sheet or a chart paper the position of Venus at its respective distance 0.73 A.U. (on a suitable scale chosen for 1 A.U.) along its corrected longitude $\lambda = 352.3°$ (increasing anti-clockwise) around Sun. Similarly we also need to plot the position of Earth on this diagram (see Fig. 1).

To locate a planet in the sky we determine its elongation ψ which is the angular distance measured eastward (that is, anti-clockwise) from Sun's position on the ecliptic, as seen from Earth (Fig. 1).

This way, from the chart, we determine the angle ψ between the line joining Earth to the planet and that from Earth to Sun. If $\psi > 0$ (measured anti-clockwise from Earth-Sun line) then the planet position lies to the east of Sun. That means that it will set after the Sun and the planet will be visible above the western horizon in the evening sky. On the other hand, if $\psi < 0$, then the planet will rise before the Sun and will be visible in the morning sky above the eastern horizon.

From Fig. 1 we find that on 1/10/2018 Venus is $\sim 33°$ east of Sun, and it will be

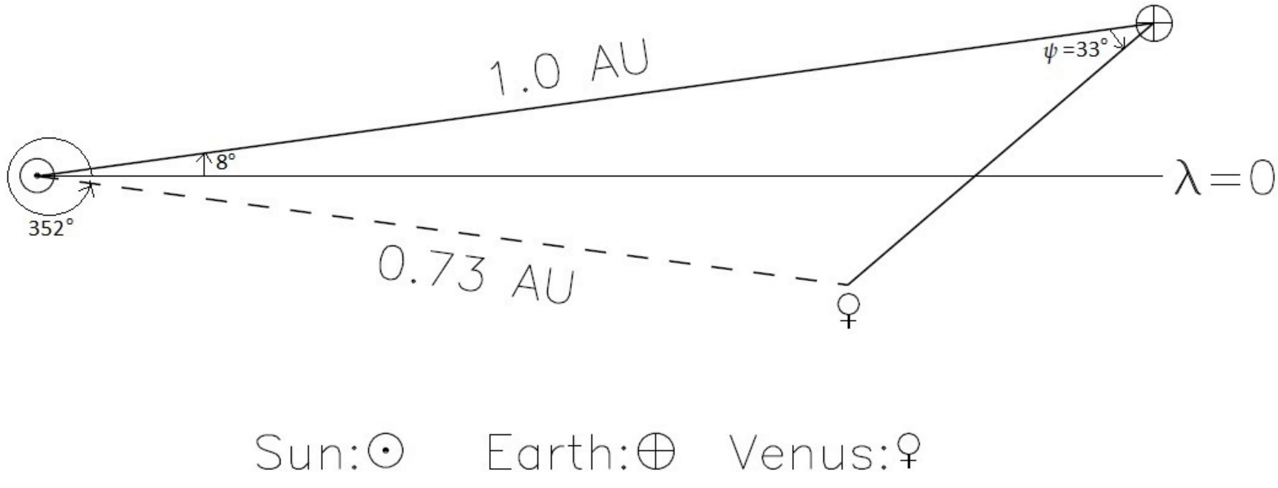

Figure 1: Elongation of Venus on 1-10-2018.

visible at sunset time about 33° away from Sun's position in the western sky.

From Fig. 2 we see that on 1/10/2018 Jupiter is $\sim$ 44° east of Sun, and it will be therefore visible in the evening about 44° away from Sun's position. Thus in the evening of 1/10/2018, one will see two bright objects (Venus and Jupiter) separated about 11° in the western sky.

As Earth completes a full rotation about its axis in 24 hours, the westward motion of the sky is at a rate $360/24 = 15°/$ hour. This is strictly true for the celestial equator, but we can use this as an approximate rotation rate even for the ecliptic, which is inclined at a 23.5° to the equator. On 1/10/2018, therefore, Venus with an eastern elongation $\sim$ 33°, will set a little more than two hours after sunset, while Jupiter at $\sim$ 44° will set slightly less than three hours after sunset.

Elongations of other planets can be calculated in the same way. In this way, one can easily locate the planets in the sky by knowing their elongations with respect to the Sun.

We may add here that all our calculations so far have been for 00:00 hr UT (Universal time) which corresponds to 05:30 hr IST (Indian Standard Time). However it is possible to determine the planetary positions for any other time of the day. For example for our cases of Venus and Jupiter, which are visible in the evening hours on our chosen date of 1/10/2018, it might be preferable to take number of days in Step 1 as 6848.5, which would then imply that our calculated positions will be for the same date but for 12:00 UT, corresponding to 17:30 hr IST, locally an evening time.

In Table 1, we have listed the elongations of all five naked-eye planets on 01-10-

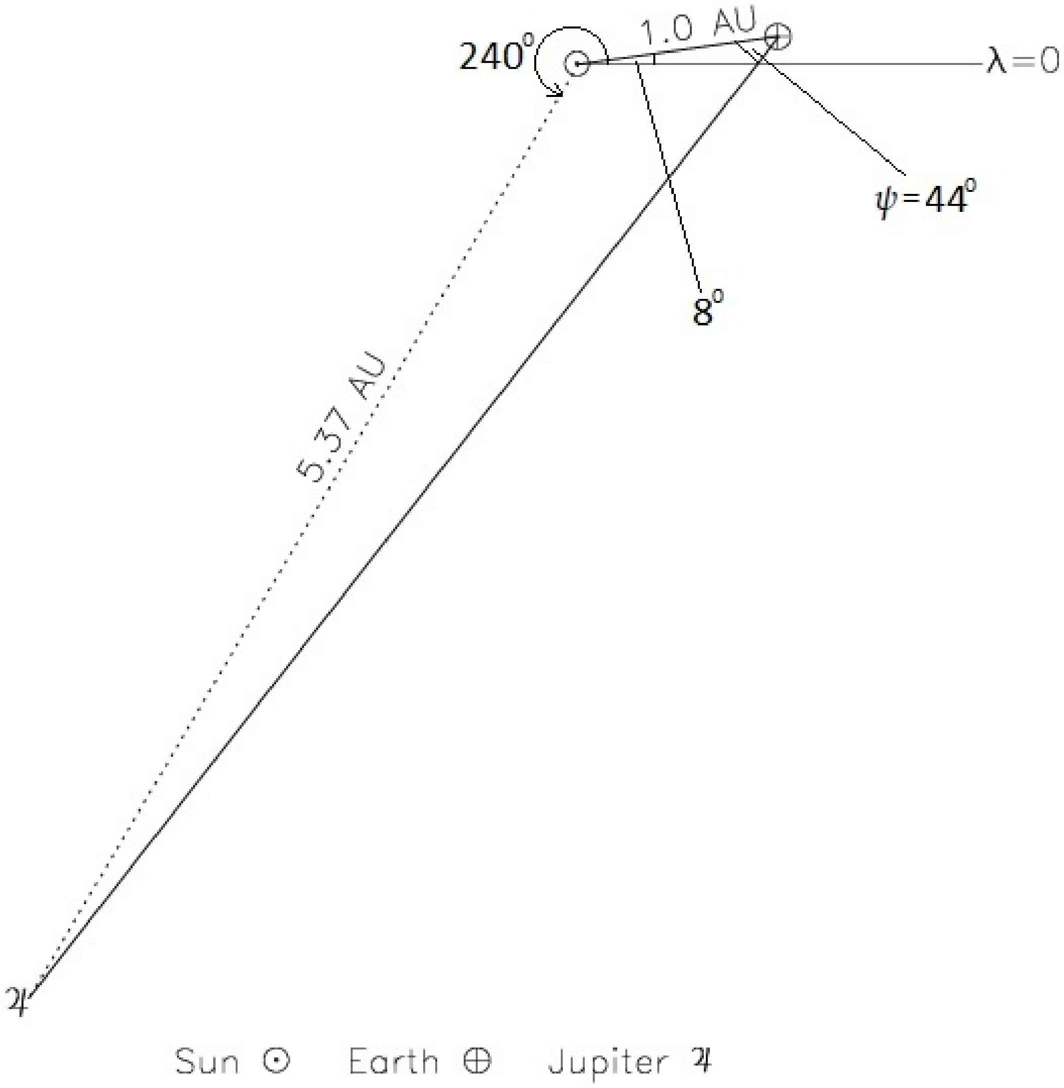

Figure 2: Elongation of Jupiter on 1-10-2018.

Table 1: Elongations of planets on 01.10.2018 at 00:00 hr and 12:00 hr UT

Planet	00:00 hr UT		12:00 hr UT	
	$\lambda(^\circ)$	$\psi(^\circ)$	$\lambda(^\circ)$	$\psi(^\circ)$
Earth	7.8		8.3	
Mercury	213.4	7.7	214.9	8.0
Venus	352.3	32.8	353.1	32.4
Mars	345.6	117.9	345.9	117.6
Jupiter	239.5	44.3	239.6	43.9
Saturn	278.9	85.4	278.9	84.9

2018 at 00:00 hr UT (5:30 IST) and at 12:00 hr UT (17:30 IST). It should be noted that not only the longitude of each planet around Sun might change by a certain amount, even the longitude of Earth advances by $\sim 1^\circ$ in a day, thus affecting the elongation of even Jupiter and Saturn (Table 1), whose angular speeds are relatively small (Table A1).

5 CONCLUSIONS

We tried to dispel a general notion that to be able to determine positions of planets in the night sky one requires complex scientific computations, using fast computers. The motive of this article has been to impress upon the reader that such accurate calculations are not really necessary for locating naked-eye planets in sky. It was demonstrated that in just three steps, one can find the positions of planets manually with simple arithmetic calculations. All one needs are the initial specifications of planetary positions for some standard epoch and the time periods of their revolutions. Then after applying a small correction for the orbital ellipticity, the location of a planet in sky, from the point of view of an Earth-based observer, could be found and thus one could get the thrill of locating a planet at the predicted position in the night sky.

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Appendix

Table A1: Parameters of the planetary orbits on 01.01.2000, 00:00 UT

Planet	λ_i ($^\circ$)	T (days)	ω_0 ($^\circ$/day)
Mercury	250.2	87.969	4.09235
Venus	181.2	224.701	1.60213
Earth	100.0	365.256	0.98561
Mars	355.2	686.980	0.52403
Jupiter	34.3	4332.59	0.08309
Saturn	50.1	10759.2	0.03346

Table A2: Correction to the longitude for elliptical orbits

λ_0	Mercury	Venus	Earth	Mars	Jupiter	Saturn
0	-24.1	-0.4	-1.7	5.0	-1.3	-6.1
10	-23.7	-0.5	-1.7	6.7	-0.2	-6.2
20	-22.4	-0.5	-1.7	8.2	0.8	-6.0
30	-20.2	-0.6	-1.6	9.4	1.8	-5.6
40	-17.1	-0.6	-1.5	10.3	2.7	-5.1
50	-13.2	-0.6	-1.3	10.8	3.6	-4.4
60	-8.6	-0.5	-1.1	11.0	4.3	-3.5
70	-3.7	-0.5	-0.9	10.8	4.9	-2.5
80	1.5	-0.4	-0.6	10.3	5.4	-1.3
90	6.6	-0.3	-0.2	9.5	5.7	-0.2
100	11.4	-0.2	0.1	8.5	5.8	1.0
110	15.6	-0.1	0.4	7.3	5.7	2.2
120	19.1	0.0	0.8	5.9	5.5	3.3
130	21.8	0.2	1.1	4.4	5.1	4.2
140	23.6	0.3	1.4	2.8	4.6	5.1
150	24.4	0.4	1.6	1.2	3.9	5.7
160	24.3	0.6	1.8	-0.4	3.2	6.2
170	23.5	0.7	2.0	-2.1	2.4	6.5
180	21.9	0.8	2.1	-3.7	1.5	6.6
190	19.8	0.9	2.1	-5.2	0.6	6.5
200	17.3	0.9	2.1	-6.6	-0.3	6.2
210	14.6	1.0	2.0	-7.9	-1.2	5.7
220	11.6	1.0	1.9	-8.9	-2.1	5.1
230	8.6	1.0	1.7	-9.8	-2.9	4.3
240	5.5	0.9	1.5	-10.3	-3.6	3.5
250	2.5	0.9	1.2	-10.6	-4.2	2.5
260	-0.6	0.8	0.9	-10.5	-4.7	1.5
270	-3.6	0.7	0.6	-10.0	-5.1	0.5
280	-6.7	0.6	0.3	-9.3	-5.3	-0.5
290	-9.7	0.5	-0.0	-8.1	-5.4	-1.5
300	-12.7	0.4	-0.4	-6.7	-5.2	-2.5
310	-15.6	0.2	-0.7	-5.0	-4.9	-3.4
320	-18.2	0.1	-0.9	-3.1	-4.5	-4.2
330	-20.5	-0.0	-1.2	-1.1	-3.8	-4.9
340	-22.4	-0.2	-1.4	1.0	-3.1	-5.5
350	-23.6	-0.3	-1.5	3.1	-2.2	-5.9

Table A3: Planetary distances (in AU) corresponding to λ_0 values

λ_0	Mercury	Venus	Earth	Mars	Jupiter	Saturn
0	0.39	0.73	1.00	1.39	4.96	9.61
10	0.37	0.73	1.00	1.41	4.95	9.52
20	0.36	0.72	1.00	1.42	4.95	9.42
30	0.34	0.72	1.00	1.45	4.96	9.33
40	0.33	0.72	0.99	1.47	4.98	9.25
50	0.32	0.72	0.99	1.49	5.00	9.18
60	0.31	0.72	0.99	1.52	5.03	9.12
70	0.31	0.72	0.99	1.54	5.07	9.07
80	0.31	0.72	0.98	1.57	5.11	9.04
90	0.31	0.72	0.98	1.59	5.15	9.02
100	0.32	0.72	0.98	1.61	5.19	9.02
110	0.33	0.72	0.98	1.62	5.24	9.05
120	0.34	0.72	0.98	1.64	5.28	9.08
130	0.35	0.72	0.99	1.65	5.32	9.14
140	0.37	0.72	0.99	1.66	5.35	9.20
150	0.38	0.72	0.99	1.66	5.39	9.28
160	0.39	0.72	0.99	1.66	5.41	9.37
170	0.41	0.72	0.99	1.66	5.43	9.46
180	0.42	0.72	1.00	1.65	5.45	9.55
190	0.43	0.72	1.00	1.64	5.45	9.64
200	0.44	0.72	1.00	1.63	5.45	9.73
210	0.45	0.72	1.01	1.61	5.44	9.81
220	0.45	0.72	1.01	1.59	5.43	9.89
230	0.46	0.72	1.01	1.57	5.41	9.95
240	0.46	0.72	1.01	1.55	5.38	10.00
250	0.47	0.73	1.01	1.52	5.35	10.04
260	0.47	0.73	1.02	1.50	5.31	10.07
270	0.47	0.73	1.02	1.47	5.27	10.08
280	0.46	0.73	1.02	1.45	5.23	10.08
290	0.46	0.73	1.02	1.43	5.19	10.06
300	0.45	0.73	1.02	1.41	5.14	10.03
310	0.44	0.73	1.01	1.40	5.10	9.98
320	0.43	0.73	1.01	1.38	5.06	9.93
330	0.42	0.73	1.01	1.38	5.03	9.86
340	0.41	0.73	1.01	1.38	5.00	9.78
350	0.40	0.73	1.01	1.38	4.97	9.70

Thermodynamics of Polar Systems

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Abstract

Equation of state for a polarized system is obtained using canonical ensemble. Energy of the system based on mean field treatment is used for the purpose [1]. The equation of state obtained matches with the van der Waals equation of state.

1 Introduction

In the limit of very low number density, all gases follow the law $PV = NkT$ where P is the pressure, V the volume, k Boltzmann's constant, T the thermodynamic temperature, and N the number of particles respectively. A gas whose behavior is governed by this equation of state over the whole range of temperatures and pressures is called an ideal gas. This equation of state can be theoretically obtained using the ensemble theory of statistical thermodynamics when there are no intermolecular forces [2, 3, 4, 5, 6, 7, 8]. At higher pressures and densities, however the equation of state of a real gas starts to deviate from ideal gas behavior because of intermolecular forces and interactions. Then the effect of interaction potential modifies the pressure and then deviate from the ideal gas nature. The net effect can be attractive or repulsive which decreases or increases the pressure and the equation of state becomes a virial series $\frac{P}{nkT} = 1 + a_2(T)n + a_3(T)n^2 + ...$, where a_l factors are known as virial coefficients [2, 3, 4, 5, 6, 7] and n is the number density. The van der Waals model [9, 10] is the first known example of an interacting system which shows phase transition, and the equation of state can be obtained by using different types of interaction potentials also using mean field approximation. van der Waals theory gives proof for the real existence of molecules and interactions and his work started with the efforts to determine the quantity which plays a peculiar role in Laplace theory of capillarity, molecular pressure and surface tension [9, 10, 11]. His firm faith in the molecular reality can be seen from his Nobel lecture "that in all my studies I was quite convinced of the real existence of molecules, that I never regarded them as a figment of my imagination, nor even as mere centers of force effects. I considered them to be the actual bodies" [11]. About the interactions van der Waals opinion was "it was necessary to determine it by means of theoretical considerations" [11]. Generally the van der Waals interaction involve an attractive and a repulsive forces between the molecules of same kind other than the bond formation. This force may be dipole dipole, dipole induced dipole, induced dipole induced dipole interactions. Here we use a dipolar system of particles and use the canonical ensemble formalism to find the equation of state.

2 Methodology

In order to get the equation of state of real gases, different potential models were used in different ensembles and more accurate methods like cluster expansion technique also is developed [2, 3]. The Gibb's paradox and extensive property of a system like entropy were studied with various external potentials from a pedagogical point of view [12]. For non ideal systems the calculation of configuration partition function which includes the position dependence is difficult to calculate with given potential in the canonical ensemble. So the method like cluster expansion was introduced by Mayer [2]. The values of the virial coefficients are calculated with the use of cluster integrals. Here we used the mean field approach to a polar system [1] and used the effective single particle potential energy. Then by using the canonical formalism the thermodynamic properties are found out.

3 Potential energy for dipolar system

Consider a system of identical molecules on a lattice with permanent electric dipole moment in the absence of any external field. The electric field due to the first dipole is given by

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{3(\hat{r_{12}}.\vec{p_1})\hat{r_{12}} - \vec{p_1}}{r_{12}^3} \tag{1}$$

In the presence of another dipole at a distance r_{12} with dipole moment $\vec{p}_2$, the interaction energy is given by $E_{12} = -\vec{p}_2.\vec{E}_1$

$$E_{12} = \frac{-1}{4\pi\epsilon_0}\frac{3(\hat{r_{12}}.\vec{p_1})(\hat{r_{12}}.\vec{p_2}) - \vec{p_1}.\vec{p_2}}{r_{12}^3} \tag{2}$$

The potential energy (ϵ_1) of the molecule 1 interacting with all other molecules in the system can be calculated using the mean field approximation by taking average over the orientations of molecule $\vec{p}_j$ by the average $\langle\vec{p}\rangle$, then

$$\epsilon_1 = -\vec{p_1}.\sum_j \frac{3(\hat{r_{1j}}.\langle\vec{p}\rangle)\hat{r_{1j}} - \langle\vec{p}\rangle}{4\pi\epsilon_0 r_{1j}^3} \tag{3}$$

$$\epsilon_1 = -\vec{p_1}.\vec{E_l} \tag{4}$$

where $\vec{E}_l$ is the local field given by

$$\vec{E}_l = \sum_j \frac{3(\hat{r_{1j}}.\langle\vec{p}\rangle)\hat{r_{1j}} - \langle\vec{p}\rangle}{4\pi\epsilon_0 r_{1j}^3} = \frac{a}{4\pi\epsilon_0}\vec{P_e} \quad (5)$$

where a is dimensionless term coming from the sample shape, $\vec{P_e} = n\langle\vec{p}\rangle$ is the polarization, and n is the number density. Since all the dipoles are of equal magnitude p

$$\epsilon_1 = -\vec{p_1}.\vec{E}_l = -cnpP_e cos\theta \quad (6)$$

where $c = \frac{a}{4\pi\epsilon_0 n} = \frac{\alpha}{n}$ with $\alpha = \frac{a}{4\pi\epsilon_0}$ and θ is the angle between $\vec{p_1}$ and $\vec{P_e}$. This single particle potential correctly predict the torque acting on the molecule 1 due to all other molecules, but this value over counts the mean value by a factor of 2. Then the suitable single particle potential is[1]

$$u_1 = \epsilon_1 - \frac{1}{2}\langle\epsilon_1\rangle \quad (7)$$

It can be shown that[1]

$$u_1 = -c\,n\,p\,P_e cos\theta + \frac{1}{2}cP_e^2 \quad (8)$$

This equation gives correct average energy criteria as $N\langle u_1\rangle = U$ with out taking the criteria of over counting of pair potential.

4 Equation of state using Canonical Ensemble

The single particle energy which is the sum of kinetic and potential energies for the polar system is given by

$$\epsilon_i = \frac{p_i^2}{2m} - cn\,pP_e\,cos\theta + \frac{1}{2}c\,P_e^2 \quad (9)$$

where p_i is the momentum. The single particle partition function is obtained using the expression

$$Q_1 = \sum_i e^{-\beta\epsilon_i}$$

Substituting the expression for energy

$$Q_1 = \frac{V'}{\lambda^3}\int_0^{2\pi} d\phi \int_0^{\pi} sin\theta d\theta e^{-\beta\left(cn\,pP_e\,cos\theta + \frac{1}{2}c\,P_e^2\right)} \quad (10)$$

where V' is the effective volume given by $V' = V - V_0$ with V_0 representing the volume of the total molecules. The N particle partition function is given by

$$Q_N = \frac{1}{N!}Q_1^N$$

The Helmholtz energy is given by

$$A = -kT\ln Q_N \quad (11)$$

Simplifying by taking $c = \frac{a}{4\pi\epsilon_0 n} = \frac{\alpha V}{N}$ we get

$$A = \frac{\alpha P_e^2 V'}{2} - NkT\ln\frac{4\pi V'}{\lambda^3 N} - NkT\ln\left[\frac{kT}{\alpha p P_e}\sinh\left(\frac{\alpha p P_e}{kT}\right)\right] \quad (12)$$

The pressure is then obtained from

$$P = -\left(\frac{\partial A}{\partial V'}\right)_{T,N} \quad (13)$$

we get

$$P = \frac{NkT}{V - V_0} - \frac{\alpha P_e^2}{2} \quad (14)$$

which gives

$$\left(P + \frac{\alpha P_e^2}{2}\right)(V - V_0) = NkT \quad (15)$$

5 Discussions and Conclusions

The non ideal equation of state for a polar system is found out using mean field approach and canonical ensemble formalism. Due to the presence of permanent dipole moment the equation state deviates from the ideal equation of state. We get the equation of state similar to the van der Waals equation of state if total polarization is taken to be proportional to the number density.

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Inertial Versus Non-Inertial Frame of Reference

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Abstract

In this paper, we have solved a rotational dynamics problem both in Inertial and Non-inertial frames of reference.
The work presents a comparison of the analysis in the two frames and comments on mathematical and physical concepts involved in solving the problem. The results are then represented in terms of simulations and solutions are thoroughly discussed, in an effort to give students, a clear picture of the importance of the two frames. The article is aimed at the undergraduate level students.

1 Introduction

Problem-Solving is the key to understanding Physics. There can be more than one way to solve a given problem and often the solutions arrived at by different approaches may not have a prima facie agreement with each other. This could lead to students remaining confused and their gradual loss of interest in problem-solving.
One of the initial steps involved in solving any problem in mechanics is to choose an appropriate frame of reference along with a suitable coordinate system to work with. Now, the frames of reference are broadly classified on the basis of their acceleration, viz, accelerated frame of reference (Non-inertial) and non-accelerated frame of reference (Inertial). Although these frames are introduced in higher secondary Physics, but their real analysis and comparison more or less remains difficult to comprehend even at undergraduate levels. Very frequently the analysis of a problem in the two frames is mixed up yielding erroneous results.
In this paper, a problem of rotational dynamics is designed and structured in such a way that after handling it in both the frames separately,the understanding of the physical parameters w.r.t. both the frames is enhanced.
Rigorous mathematics is involved while analysing the problem in inertial frame

using both the Cartesian and the Spherical polar coordinates. However, it gives an insight of the physical processes involved. The trajectory of the bead so obtained is illustrated to convey complete information about its motion. Contrary to this, the tedious mathematics involved in the inertial frame can be bypassed and the solution can be obtained in a much simpler manner in the non-inertial frame. Again, the trajectory in this frame is also illustrated and discussed. A detailed comparison of the analysis of the given problem in both the frames is also done.

Problem Statement:

A bead of mass 'm' slides without friction on a rigid rod rotating at constant angular speed ω , the rod being inclined at an angle α with the axis of rotation, as shown in Figure 1. The problem is to find the trajectory of the Bead and compare the results in both the inertial and non-inertial frames.Assume the bead to be initially at rest, at a distance r_0 from the pivoted end.

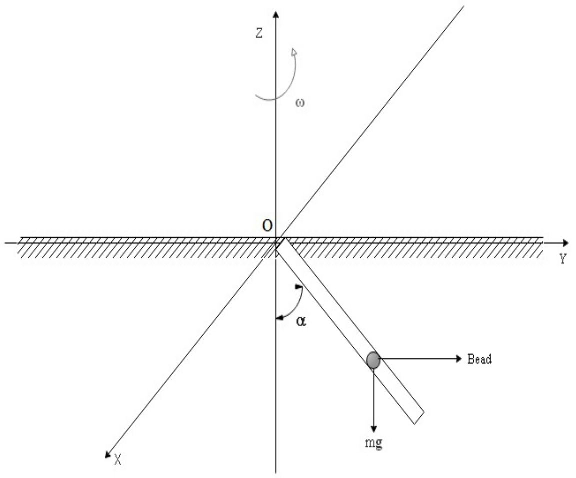

Figure 1:Bead sliding along a rotating rigid rod. The angular velocity $\vec{\omega}$ is along the positive z axis, mg is the gravitational force acting vertically downwards on the bead.

2 Non-Inertial Frame: A frame of reference attached to the rotating rod

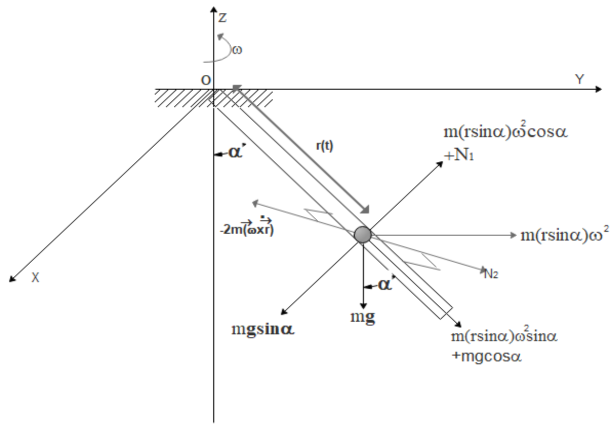

Figure 2: Forces acting on the bead as viewed by an observer in the Non- Inertial Frame

Figure 2 shows the various forces acting on the bead as viewed by an observer in the Non- Inertial Frame. The rod is pivoted at O. $r(\vec{t})$ is the instantaneous position vector of the bead along the rod w.r.t the pivoted end and is directed away from the pivot. The forces acting on the bead are the normal contact forces N_1 and N_2 in two different planes perpendicular to each other; the Coriolis force [-2m($\vec{\omega} \times \dot{\vec{r}}$)] acting perpendicular to the rod in a plane perpendicular to the plane containing $\vec{\omega}$ and $\vec{r}$ (the rod); the gravitational force, mg, acting vertically downwards; the centrifugal force, m$(r\sin\alpha)\omega^2$ acting radially outwards perpendicular to the axis of rotation.

At any instant, if $\vec{\omega}$ and the rod are in the plane of the paper, i.e., the YZ-plane, then the Coriolis force acts perpendicular to the rod along the X axis. It is balanced by the normal force, N_2 , from the rod on the bead in the opposite (-X) direction. At this instant, the component of the centrifugal force, $m(rsin\alpha)\omega^2\cos\alpha$, the component of gravitational force, $mg\sin\alpha$, both acting on the bead perpendicular to the length of the rod and the Normal contact force (N_1) experienced by the bead from the rod, all lie in the plane of the paper (the YZ-plane). Since the motion of the bead is along the rod, as seen by the non-inertial observer, there is no net force acting on the bead perpendicular to the length of the rod. So, we can write,

$$mg\sin\alpha = N_1 + m(r\sin\alpha)\omega^2\cos\alpha$$

At all instants of time, the bead experiences no net force in any direction other than the one along the rod.

If $r(\vec{t})$ is the instantaneous position vector of the bead along the rod with respect to the pivoted end, then, by Newton's second law, the equation of motion of the bead is:

$$m\frac{d^2\vec{r}}{dt^2} = (m\omega^2 r\sin^2\alpha + mg\cos\alpha)\hat{r}$$

The first term on the RHS of equation (2.2) is the component of centrifugal force along the rod and second term is the gravitational force component along the rod. The corresponding scalar equation is:

$$\ddot{r} = \omega^2 r\sin^2\alpha + g\cos\alpha$$

This is a second order ,linear, non-homogeneous differential equation that can be solved by the method of undetermined coefficients.

Solution of the differential equation:

The characteristic equation is:

$\Lambda^2 - \omega^2\sin^2\alpha = 0$

The roots are $\Lambda_1 = +\omega\sin\alpha$

and $\Lambda_2 = -\omega\sin\alpha$

Hence the solution of the homogeneous part is

$$r = Ae^{\omega\sin\alpha t} + Be^{-\omega\sin\alpha t}$$

where, A B are arbitrary constants.

For the particular solution, using method of undetermined coefficient, let r(t)=c (constant)

Substituting this in equation (2.2) We get,

$$0 = g\cos\alpha + c\omega^2\sin^2\alpha$$
$$c = -\frac{g\cos\alpha}{\omega^2\sin^2\alpha}$$

The complete solution is therefore

$$r(t) = Ae^{\omega\sin\alpha t} + Be^{-\omega\sin\alpha t} - \frac{g\cos\alpha}{\omega^2\sin^2\alpha}$$

Using the initial conditions,

At t=0 , $r = r_0$

$r_0 = A + B - \gamma \quad where \quad \gamma = \frac{g\cos\alpha}{\omega^2\sin^2\alpha}$

At t=0, $\dot{r} = 0$

$A - B = 0$

Therefore, $A = B = \frac{r_0+\gamma}{2}$
Substituting the above values we get,
$r(t) = (r_o + \gamma)\cosh((\omega \sin\alpha)t) - \gamma$
This is the equation of trajectory of the bead in the non- inertial frame. The non- inertial observer sees the bead moving in a straight line along the rod.
The Scilab plot[4] of equation (2.4) is shown in figure 3.

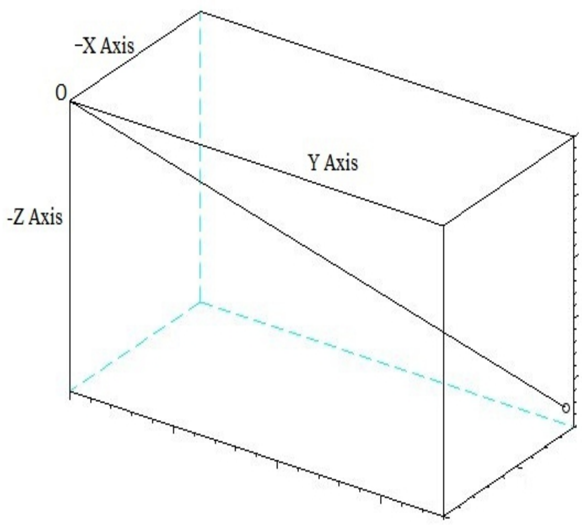

Figure 3: Linear trajectory of the bead in the Non- Inertial frame

From the above analysis of the problem in the non-inertial frame of reference, we can find the Normal contact forces acting on the bead due to the rod as follow:

$$N_1 = mg\sin\alpha - m(r\sin\alpha)\omega^2\cos\alpha$$
$$N_2 =\mid -2m(\vec{\omega}\times\dot{\vec{r}}) \mid = 2m\omega\dot{r}\sin\alpha$$

3 Inertial Frame:A frame of reference with respect to which the rod is rotating at an angular velocity $\vec{\omega}$ making an angle α with the rod

3.1 Cartesian Coordinates

The Cartesian system of coordinates is most often our default choice in problem solving and we are, somehow, hardwired to use it. The same is done here. Referring to Figure 4, $r(\vec{t})$ is the instantaneous position vector of the bead along the rod w.r.t the pivoted end and directed away from the pivot. So, the instantaneous cartesian coordinates of the bead are:

$$x(t) = r(t)\cos\omega t\sin\alpha$$
$$y(t) = r(t)\sin\omega t\sin\alpha$$
$$z(t) = -r(t)\cos\alpha$$

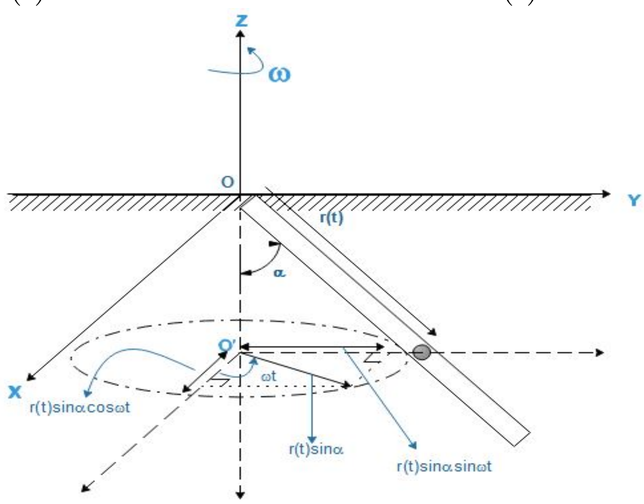

Figure 4: Orientation of the rod in cartesian coordinates. $r(\vec{t})$ is the instantaneous position vector of the bead from the pivot

O. $r(t)\sin\alpha$ is the instantaneous radius of curvature of the path of the bead. $r(t)\sin\alpha\cos\omega t$ and $r(t)\sin\alpha\sin\omega t$ are the instantaneous x(t) and y(t) coordinates of the bead respectively.

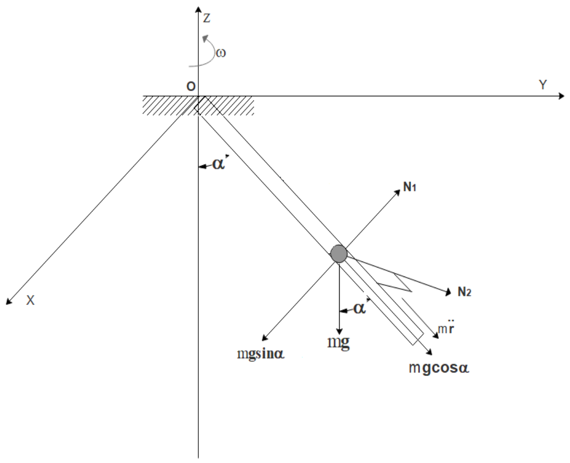

Figure 5: Forces acting on the bead as observed by an observer in the Inertial frame.

Figure 5 shows the various forces acting on the bead as observed in the inertial frame. The bead experiences the gravitational force, mg, acting vertically downwards and the Normal contact forces N_1 and N_2 in two different planes perpendicular to each other. $mg\cos\alpha$ and $mg\sin\alpha$ are the components of the gravitational force along the rod and perpendicular to the rod respectively.

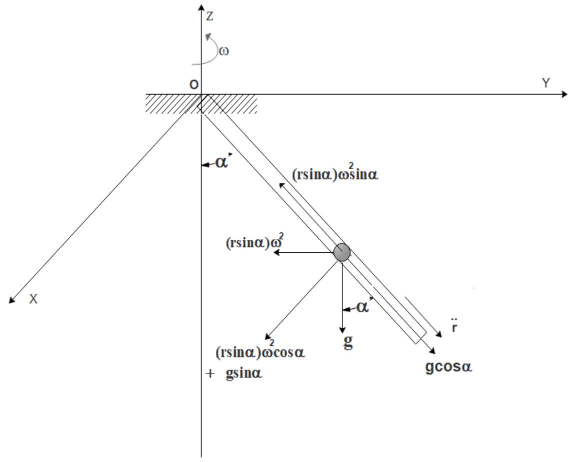

Figure 6: Acceleration experienced by the bead observed by an observer in the Inertial frame

Figure 6 shows the various components of accelerations experienced by bead during its motion.The bead experiences a centripetal acceleration, $r\sin\alpha\omega^2$, acting radially inwards towards the axis of rotation. $(r\sin\alpha\omega^2)\sin\alpha$ and $(r\sin\alpha\omega^2)\cos\alpha$ are the components of centripetal acceleration experienced by the bead along the rod and perpendicular to the rod respectively. $\ddot{r}$ is the instantaneous acceleration of the bead along the rod, directed away from the pivot.

The component of gravitational force, $mg\cos\alpha$, experienced by the bead along the rod is partly used to provide the necessary centripetal acceleration component, $(r\sin\alpha)\omega^2\sin\alpha$, to the bead along the rod and the remaining part of the gravitational force component accelerates it down the rod away from the pivot. Hence, the expression for the acceleration of the bead along the rod is:

$$[\ddot{r} - (r \sin\alpha)\omega^2 \sin\alpha]\hat{r} = g\cos\alpha\hat{r}$$

where, $\hat{r}$ is the unit vector along $r(\vec{t})$.
The corresponding scalar equation can be written as:

$$\ddot{r} = g\cos\alpha + \omega^2 r \sin^2\alpha$$

The equation (3.3) is identical to equation (2.3).

So, substituting the expression for r(t) obtained in equation (2.4) in equations (3.1), we get the parametric equations of the trajectory of the bead as observed in the inertial frame. On plotting the equations (3.1) alongwith equation (2.4) using Scilab [4], the trajectory of the bead comes out to be helical with a changing radius, as shown in Figure 7.
Thus, we see that both the inertial and the non-inertial observers agree on the values of the acceleration experienced by the bead along the length of the rod [Equations 3.3 and 2.3]. However, unlike the non-inertial observer, the motion of the bead is not linear. Since, the trajectory of the bead is helical in the inertial frame of reference [Figure 7], the inertial observer sees additional components of acceleration other than the component of acceleration $\ddot{r}$, of the bead along the rod.
To determine these components of acceleration we need to write down the equations of motion of the bead in directions normal to the rod.
One of these equations is quite obvious, i.e.,

$$(mg\sin\alpha - N_1) = m(r\sin\alpha\omega^2\cos\alpha)$$

Thus, we get the component of centripetal acceleration $[(r\sin\alpha\omega^2)\cos\alpha]$ experienced by the bead perpendicular to the length of the rod.
The Normal force N_2 acting on the bead also accelerates it. This component of acceleration can be obtained by dividing the magnitude of the normal force N_2 in equation (2.5) with the mass of the bead.
This component of acceleration comes out to be equal to $2\omega\dot{r}\sin\alpha$. It is obtained by dividing the magnitude of the normal force in equation (2.5) with mass of the bead.

$$N_2/m = 2\omega\dot{r}\sin\alpha$$

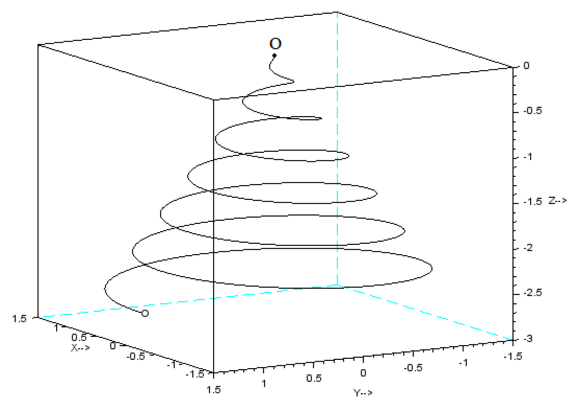

Figure 7: Helical trajectory of the bead as observed in the Inertial frame. It has an increasing radius as the bead slides along

the rod away from the pivoted end.

3.2 Spherical Polar Coordinates

Alternatively, the motion of the bead, in the inertial frame can be analysed using Spherical polar coordinates which quite simplifies the mathematics involved in the analysis.

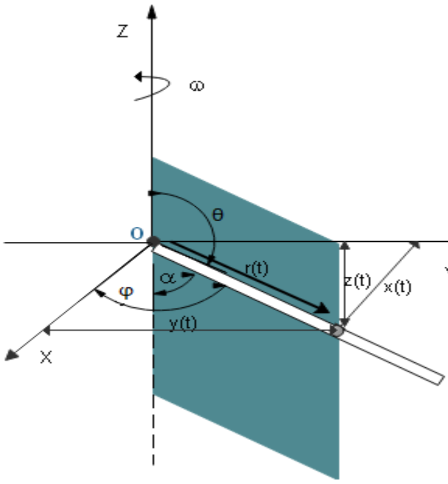

Figure 8: Orientation of the rod in Spherical Polar Coordinates

Figure 8 shows the orientation of the rod in Spherical polar coordinates. The relation between the Spherical polar and Cartesian coordinates is given by:

$$x(t) = r(t)\sin\theta\cos\phi$$
$$y(t) = r(t)\sin\theta\sin\phi$$
$$z(t) = r(t)\cos\theta$$

where, θ is the polar angle $[0 \leq \theta \leq \pi]$ and ϕ is the azimuthal angle $[0 \leq \phi \leq 2\pi]$.

On mapping the spherical coordinates with our problem, we have:

$$\theta = \pi - \alpha$$
$$\phi(t) = \omega t$$

Since α is a constant as the rod is inclined at a fixed angle, θ also does not change with time. The rate of change of the azimuthal angle ϕ is equal to the angular speed of rotation of the rod, ω, and is also a constant.

The expression for acceleration in spherical polar coordinates is:

$$\ddot{\vec{r}} = (\ddot{r} - r\omega^2\sin^2\theta - r\dot{\theta}^2)\hat{e_r} + (r\dot{\omega}\sin\theta + 2\dot{r}\omega\sin\theta + 2r\omega\dot{\theta}\cos\theta)\hat{e_\phi} + (r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\omega^2\sin\theta\cos\theta)\hat{e_\theta}$$

where, $\hat{e_r}, \hat{e_\phi}$ and $\hat{e_\theta}$ are the radial, azimuthal and polar unit vectors respectively. On mapping with our ongoing analysis, $\hat{e_r}$ is identical to $\hat{r}$.

So, the radial equation of motion is:

$$m[\ddot{r} - r\omega^2\sin^2\theta - r\dot{\theta}^2] = mg\cos\alpha$$

Using equations (3.7) the above equation reduces to:

$$\ddot{r} = g\cos\alpha + r\omega^2\sin^2\alpha \qquad \textbf{3.10}$$

As expected, the equation (3.10) is identical to equation (3.3) and (2.3).

On going back to equation (3.8), we see

that it explicitly gives the other components of acceleration experienced by the bead in the inertial frame. We may refer to the component of acceleration along $\hat{e_\theta}$ as the "polar acceleration", a_θ, of the bead and the component along $\hat{e_\phi}$ as the "azimuthal acceleration", a_ϕ, of the bead. These are given by:

$$a_\phi \hat{e_\phi} = 2\dot{r}\omega \sin\theta \hat{e_\phi}$$
$$a_\theta \hat{e_\theta} = -r\omega^2 \sin\theta \cos\theta \hat{e_\theta}$$

(Since, $\dot{\theta} = 0$ and $\dot{\omega} = \ddot{\phi(t)} = 0$, using $\theta = (\pi - \alpha)$)
The scalar form of above equations can be re-written as:

$$a_\phi = 2\dot{r}\omega \sin\alpha$$
$$a_\theta = r\omega^2 \sin\alpha \cos\alpha \qquad \textbf{3.12}$$

The so termed "polar acceleration" can be identified with the component of centripetal acceleration experienced by the bead normal to the rod. The "azimuthal acceleration" can be identified with the acceleration experienced by the bead due to the Normal contact force N_2 acting on it [as shown in the equation (3.5)].

Torque and Angular Momentum:

If $\vec{L}$ is the angular momentum of the bead about the pivot, then we can write

$$\vec{L} = \vec{r} \times m\dot{\vec{r}}$$

Differentiating the above equation w.r.t. time, we get the rate of change of angular momentum of the bead as,

$$\dot{\vec{L}} = \dot{\vec{r}} \times m\dot{\vec{r}} + \vec{r} \times m\ddot{\vec{r}}$$

which reduces to

$$\dot{\vec{L}} = \vec{r} \times m\ddot{\vec{r}}$$

Using $\vec{r} = r\hat{e_r}$ and substituting for $\ddot{\vec{r}}$ from equation 3.8, the expression for $\dot{\vec{L}}$ becomes,

$$\dot{\vec{L}} = r\hat{e_r} \times m[(\ddot{r} - r\omega^2 \sin^2\theta)\hat{e_r} + (2\dot{r}\omega \sin\theta)\hat{e_\phi} - (r\omega^2 \sin\theta \cos\theta)\hat{e_\theta}]$$

$$\dot{\vec{L}} = mr[2\dot{r}\omega \sin\theta(\hat{e_r} \times \hat{e_\phi}) - r\omega^2 \sin\theta \cos\theta(\hat{e_r} \times \hat{e_\theta})]$$

$$\dot{\vec{L}} = mr[2\dot{r}\omega \sin\theta(-\hat{e_\theta}) - r\omega^2 \sin\theta \cos\theta(\hat{e_\phi})]$$

Since $\theta = \pi - \alpha$,

$$\dot{\vec{L}} = mr(-2\dot{r}\omega \sin\alpha \hat{e_\theta} + r\omega^2 \sin\alpha \cos\alpha \hat{e_\phi})$$

If $\vec{\tau}$ is the torque experienced by the bead about the pivot, then,

$$\vec{\tau} = \dot{\vec{L}}$$

Therefore,

$$\vec{\tau} = r[-2m\dot{r}\omega \sin\alpha \hat{e_\theta} + m(r\sin\alpha)\omega^2 \cos\alpha \hat{e_\phi}]$$

The negative sign in the first term shows that this component of torque acts to decrease the angular momentum of the bead. We re-write the above equation as

$$\vec{\tau} = -\tau_\theta \hat{e_\theta} + \tau_\phi \hat{e_\phi}$$

, where,

$$\tau_\theta = r2m\dot{r}\omega \sin\alpha$$
$$\tau_\phi = rm(r\sin\alpha)\omega^2 \cos\alpha$$

The τ_ϕ component can be shown to be equal to the resultant torque experienced by the bead about the pivot due to the gravitational force component, $mg\sin\alpha$, and the normal contact force N_1.
$\tau_\phi = r(mg\sin\alpha - N_1)$

Substituting for N_1 from equation (2.5) in equation (3.20), we get

$$\tau_\phi = rm(r\sin\alpha)\omega^2 \cos\alpha$$

which is identical to its expression in equation (3.19).

Similarly, the τ_θ component can be shown to be equal to the torque experienced by the bead about the pivot due to the Normal contact force N_2 acting on it.

$$\tau_\theta = rN_2$$

Substituting for N_2 from equation (2.5), we get,

$$\tau_\theta = r2m\dot{r}\omega \sin\alpha$$

which is identical to its expression in equation (3.19).

4 RESULTS AND DISCUSSIONS

Arriving at a detailed solution of the motion of the bead, equations of trajectories are obtained in both the frames.The non-inertial observer sees the bead moving along the rod, away from the pivot,i.e,the trajectory of the bead is linear[Figure 3].On the other hand, the inertial observer sees the rod rotating, and the bead sliding along it away from the pivot. The trajectory of the bead in the inertial frame is helical with increasing radius [Figure 7].
Both the observers agree on the acceleration experienced by the bead along the rod and rightly so. However,the inertial observer also sees the bead accelerating in other directions in addition to the one along the rod. Having a closer look at equation (3.12), we see that a_ϕ multiplied by the mass of the bead corresponds to the Coriolis force acting on the bead in the non-inertial frame.Similarly, a_θ multiplied by mass of the bead corresponds to the component of centrifugal force experienced by the bead normal to the rod in the non-inertial frame.
This illustrates an important concept:
"The only difference in writing equation of motion in the two frames is that the acceleration term in the inertial frame turns into a

fictitious force term in the accelerating frame and appears on the other side of the equation”.

It can be seen that the analysis of trajectory of the bead in the inertial frame requires quite an elaborate mathematical skill.The mathematics is somewhat simplified when we use spherical polar coordinates to analyze acceleration of the bead and other dynamical aspects of its motion like torque and angular momentum in the inertial frame.

The advantage of tackling the problem in the non-inertial frame lies in the ease of evaluating acceleration of the bead along the length of the rod, in terms of the mathematical complexity. The determination of the Normal Contact forces is also quite straightforward in the non-inertial frame.

5 Conclusion

The comparison of the analysis of the given problem in both inertial and non-inertial frames highlights the significance of both frames with the pivotal elements of each in focus. The choice of a reference frame could be made on the basis of what aspect of the problem is to be analyzed. The present work attempts to reveal the advantage of using a suitable coordinate system that makes use of the symmetry (if any) of a given problem. This goes a long way in easing out the mathematics needed to solve the problem.

6 References

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A Heuristic Derivation of Radiative Power Loss and Radiation Reaction from the Kinetic Power of Electric Inertial Mass of a Charge

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Abstract

It is shown that formulas for the radiative power loss and radiation reaction from a charge can be derived in a heuristic manner from the kinetic power (rate of change of the kinetic energy) of its electric inertial mass. The derivation assumes a non-relativistic but otherwise an arbitrary motion of the charge. We exploit the fact that as the charge velocity changes because of a constant acceleration, there are accompanying modifications in its electromagnetic fields which can remain concurrent with the charge motion because the velocity as well as acceleration information enters into the field expression. However, if the acceleration of the charge is varying, information about that being not present in the field expressions, the electromagnetic fields get "out of step" with the actual charge motion. Accordingly we arrive at a radiation reaction formula for an arbitrarily moving charge, obtained hitherto in literature from the self-force, derived in a rather cumbersome way from the detailed mutual interaction between various constituents of a small charged sphere. This way we demonstrate that a power loss from a charge occurs only when there is a change in its acceleration and the derived instantaneous power loss is directly proportional to the scalar product of the velocity and the rate of change of the acceleration of the charge.

1 Introduction

Radiation reaction was proposed first by Lorentz [1, 2] and later Abraham [3] and Lorentz [4] derived it in detail for an arbitrarily moving small charged sphere, and is now available in various forms in many textbooks [5, 6, 7, 8, 9, 10, 11]. The formula, hitherto obtained in literature in a rather cumbersome way, is evaluated from the detailed mutual interaction between constituents of the charged sphere. The self-force turns out to be proportional to the rate of change of acceleration, independent of the radius of the small sphere. One obtains the instantaneous radiative power loss formula by a scalar product of the self-force with the velocity of the charge. The same formula for the radiative power loss is also obtained from the Poynting flux in the neighbourhood of a “point charge” in arbitrary motion [12]. Further, the radiation reaction formula has been derived also from the rate of electromagnetic momentum flow, calculated using the Maxwell stress tensor, across a surface surrounding the neighbourhood of a point charge, [13].

However this power loss formula does not agree with the standard Larmor’s formula, where one calculates the Poynting flux through a spherical surface of large enough radius r centred on the time-retarded position of the charge. The flux turns out to be proportional to the square of acceleration ($\propto \dot{\mathbf{v}}^2$) [5, 6, 8].

There is extensive literature on this controversy of which of these two formulas gives correct description of radiative losses [14, 15, 16, 17]. Larmor’s formula leads to wrong conclusions in the instantaneous rest-frame of an accelerated charge, where the charge has no velocity and thus no kinetic energy to be lost into radiation. On the other hand Larmor’s formula predicts a continuous radiative loss proportional to the square of acceleration even for an instantly stationary charge. Often an extraneous acceleration-dependent term called Schott energy is introduced to make the two formulas conform to each other [9, 18, 19, 20, 21, 22]. But recently it has been explicitly shown that the Poynting flux passing through a spherical surface of vanishingly small dimensions surrounding the charge, in its instantaneous rest-frame, is zero [12]. Actually in all neighbourhood of the charge in its instantaneous rest-frame, the transverse terms of the time-retarded velocity fields cancel the acceleration fields, which were responsible in Larmor’s formula for radiation. This removes the need for the acceleration-dependent Schott-energy term, introduced in the literature on an ad hoc basis to comply with law of energy conservation (see also [23, 24, 25]).

The radiation reaction formula could also be derived from Larmor’s formula of radiative losses using the law of energy conservation [5, 6, 8], but only if the Schott term remains unchanged at the ends of the time interval considered, which happens if the motion is cyclic. Momentum conservation also remains a problem in Larmor’s radiation picture. The radiation pattern of an accelerated charge has a $\sin^2\phi$ dependence about the direction of acceleration [5, 6, 8]. Due to this azimuthal symmetry the net momentum carried by the radiation is nil. Therefore the charge too cannot be losing momentum, even though it is undergoing radiative losses. Thus we have a paradox of a radiating charge losing its kinetic energy but without a corresponding change in its linear momentum. An effect of this inconsistency appears in a synchrotron radiation case, where Larmor's formula leads to conclusions about the dynamics of the radiating charge that are not conversant with special relativity and violate energy-momentum conservation[26]. However no such inconsistencies arises when one makes use of the radiation reaction formula to calculate energy-momentum losses of the radiating charge [27].

More recently it has been shown that the two formulas are compatible and no controversy really arises if one keeps a proper distinction between the retarded time and the real time [28]. In particular, one gets Larmor's formula, with radiative losses proportional to the square of the acceleration if one expresses the radiated power in terms of quantities describing motion of the charge at the retarded time. On the other hand if the motion of the charge is expressed in terms of real time ("present") quantities, then one arrives at the power loss formula usually derived from the radiation reaction formulation, i.e., the radiative power loss proportional to the scalar product of the velocity and the first time derivative of the acceleration of the charge.

Without going any further into the controversy between the two radiation loss formulas, here we show that the radiation reaction formula can be derived in an alternate, though heuristic, method, from the mechanical motion of the charge if one takes the electrical mass of the charge as its inertial mass. For this we shall make use of the kinetic power, i.e., a temporal rate of change of the kinetic energy, of the charge to derive a formula for radiative losses of the charge.

2 Electric inertial mass of a charge

The electromagnetic field momentum is given by the volume integral [5]

$$\mathbf{P}_{\rm field} = \frac{1}{4\pi c}\int {\rm d}V\,(\mathbf{E}\times\mathbf{B}). \qquad (1)$$

A charge, assumed to be a uniformly charged spherical shell of radius $r_{\rm o}$, moving with a non-relativistic, uniform velocity $\mathbf{v}_{\rm o}$, from the above volume integral, possesses an electromagnetic field momentum

$$\mathbf{P}_{\rm field} = \frac{2e^2\mathbf{v}_{\rm o}}{3r_{\rm o}\,c^2} = m_{el}\mathbf{v}_{\rm o}\,, \qquad (2)$$

with an electric mass defined as $m_{el} = 4U_0/3c^2$ [29], where $U_0 = e^2/2r_{\rm o}$ is the energy in self-fields of the charge in its rest frame. The factor of 4/3 in the inertia of electric mass has long since been highly annoysome. Poincaré [30] pointed out that in a real charged particle, there must be some non-electrical (!) "binding" forces to balance the Coulomb self-repulsion of the charge, which would remove the factor of 4/3. However these *non-electrical* binding forces are not represented in the expressions of the *electromagnetic* fields and an explanation for this factor of 4/3 must be found within the electromagnetic theory itself. It has been explicitly shown [31] that this extra factor in the expression for the total electromagnetic momentum of the charge arises because of the energy flow associated with the electromagnetic self-repulsion force within the charge constituents. The net force on one hemisphere of the charge is along the direction of motion, and on the remaining hemisphere it is in a direction opposite to the motion. Therefore as the charge moves, a positive work is being done by self-force *on* the forward hemisphere, while an equal amount of work is being done *by* the backward hemisphere against the self-force. Though there is no net increase in the energy of the total system, yet because of the electromagnetic self-force there is a continuous flow of energy across the charged sphere between its two halves, implying a corresponding momentum due to this energy-flow. This momentum is important even for non-relativistic velocities and gives 1/3rd additional contribution to the otherwise momentum of the charge [31], thereby explaining this intriguing factor of 4/3 in the total electromagnetic momentum.

3 Electromagnetic field momentum of a uniformly accelerated charge

Electromagnetic field of a charge e, from the laws of electrodynamics, is determined at time t by the charge motion ($\mathbf{v}$ and $\dot{\mathbf{v}}$) at the retarded time $t' = t - r/c$ [5, 6, 8, 32].

$$\mathbf{E} = \left[\frac{e(\mathbf{n} - \mathbf{v}/c)}{\gamma^2 r^2 (1 - \mathbf{n}\cdot\mathbf{v}/c)^3} + \frac{e\mathbf{n} \times \{(\mathbf{n} - \mathbf{v}/c) \times \dot{\mathbf{v}}\}}{rc^2\,(1 - \mathbf{n}\cdot\mathbf{v}/c)^3}\right]_{t'} \tag{3}$$

$$\mathbf{B} = \mathbf{n} \times \mathbf{E} \tag{4}$$

The first term on the right hand side of (Eq. (3)) that fall with distance as $1/r^2$, is called velocity fields while the second term, falling with distance as $1/r$, is called the acceleration fields, the latter generally assumed to be solely responsible for radiation from the charge. It is a standard practice to assign the Poynting flux, calculated using acceleration fields, through a spherical surface, say Σ, of radius $r = c(t - t')$, centred on the charge position at the retarded time t', as the radiation losses by the charge at time t', to get Larmor's formula for radiative losses [5, 6, 8]. However, Poynting theorem tells us that the rate of the kinetic energy loss by charge at *present time* $t = t' + r/c$ (and not at retarded time t') is related to the instantaneous outgoing electromagnetic power (Poynting flux) at t from the surface Σ [5, 6, 8]. It may though be recalled that the fields at the surface Σ are determined by the motion of the charge at the retarded time t' (Eq. (3)). This might appear to be a break down of causality, after all, how come the Poynting flux determined from the motion of the charge in past, i.e., at an earlier time t', is being equated to the kinetic energy-loss rate of the charge at a later time t? How can one be sure that the charge will not behave erratic between t' and t, thus while keep-ing the Poynting flux unaffected (which is already decided by the charge motion at t') but modifying the kinetic power loss rate of the charge? Actually, even the charge motion at the present time t follows from the charge motion at t', determined by the laws of mechanics, and thus both the electromagnetic fields on the surface Σ as well as the charge motion at time t are determined by the charge motion at t' and there is no conflict with the causality.

Let us first consider the case of a uniform acceleration $\dot{\mathbf{v}}$. The charge motion due to acceleration is,

$$\mathbf{v}_\mathrm{o} = \mathbf{v} + \dot{\mathbf{v}}(t_\mathrm{o} - t') \tag{5}$$
$$\dot{\mathbf{v}}_\mathrm{o} = \dot{\mathbf{v}}\,, \tag{6}$$

were $\mathbf{v}$, $\dot{\mathbf{v}}$ on the right hand side represent respectively the velocity and acceleration of the charge at the retarded time t', while $\mathbf{v}_\mathrm{o}, \dot{\mathbf{v}}_\mathrm{o}$ on the left hand side represents the corresponding values at the present time t.

For our considerations, we assume the charge motion to be non-relativistic, where Lorentz contraction may not play any role and the moving charge continues to be a uniformly charged sphere of radius r_o, as when it is at rest.

Using the vector identity $\mathbf{v} = \mathbf{n}(\mathbf{v}.\mathbf{n}) - \mathbf{n} \times \{\mathbf{n} \times \mathbf{v}\}$ in the expression for electric field (Eq. (3)), the electric field for a non-relativistic motion of the charge, thereby dropping all terms which are non-linear in $\mathbf{v}$ or its derivatives, can be written as

$$\mathbf{E} = \frac{e\,\mathbf{n}}{r^2(1-\mathbf{n}\cdot\mathbf{v}/c)^2} + \frac{e\mathbf{n}\times(\mathbf{n}\times\mathbf{v})}{cr^2} + \frac{e\mathbf{n}\times(\mathbf{n}\times\dot{\mathbf{v}})}{c^2 r}\,. \tag{7}$$

Now, in the case of a uniform acceleration, the *retarded value* of the velocity will be $\mathbf{v} = \mathbf{v}_\mathrm{o} - \dot{\mathbf{v}}r/c$ (Eq. (5)). Then Eq. (7) for the electric field becomes

$$\mathbf{E} = \frac{e\mathbf{n}}{r^2}\left[1 + \frac{2\mathbf{n}\cdot\mathbf{v}_\mathrm{o}}{c} - \frac{2\mathbf{n}\cdot\dot{\mathbf{v}}r}{c^2}\right] + \frac{e\mathbf{n}\times(\mathbf{n}\times\mathbf{v}_\mathrm{o})}{cr^2}\,, \tag{8}$$

with the magnetic field given by

$$\mathbf{B} = \frac{-e\,\mathbf{n}\times\mathbf{v}_\mathrm{o}}{r^2 c}\,. \tag{9}$$

This begets for the Poynting flux

$$\mathcal{S} = \frac{c}{4\pi}\oint_\Sigma \mathrm{d}\Sigma\,\mathbf{n}\cdot(\mathbf{E}\times\mathbf{B}) = \frac{e^2\mathbf{v}_\mathrm{o}^2}{2r^2c}\int_\mathrm{o}^\pi \mathrm{d}\theta\,\sin^3\theta = \frac{2e^2\mathbf{v}_\mathrm{o}^2}{3r^2c}\,. \tag{10}$$

In the case of a uniformly accelerated charge, evidently, there is no term proportional to $\dot{\mathbf{v}}^2$, independent of r, which is usually called the radiated power. Instead, the Poynting flux (Eq. (10)) is merely what would be for a hypothetical charge moving with a uniform velocity $\mathbf{v}_\mathrm{o}$, which is nothing but the velocity of the actual charge at the present time.

Now in the instantaneous rest-frame of the charge, $\mathbf{v}_\mathrm{o} = 0$, which means Poynting flux is zero (Eq. (10)). In fact, everywhere, the transverse component of the electric field is zero, and so is the magnetic field. Incidentally Pauli [33] first pointed it out that magnetic field is throughout zero in the instantaneous rest-frame of a uniformly accelerated charge, indicating the absence of radiation from a uniformly accelerated charge.

We can substitute for **E** and **B** from Eqs. (8) and (9) in Eq. (1) to calculate the electromagnetic field momentum of a uniformly accelerated charge, having a non-relativistic motion. The transverse component of the electric field makes a nil contribution to the volume integral in Eq. (1). In fact, the only finite contribution to the electromagnetic field momentum comes from the first radial term $(e\mathbf{n}/r^2)$ in Eq. (8) to yield

$$\begin{aligned}\mathbf{P}_{\rm field} &= \frac{-e^2}{4\pi c}\int {\rm d}V\, \frac{\mathbf{n}\times(\mathbf{n}\times\mathbf{v}_{\rm o})}{r^4 c}\\ &= \frac{e^2\mathbf{v}_{\rm o}}{2c^2}\int_{\rm o}^{\pi}{\rm d}\theta\,\sin^3\theta\int_{r_{\rm o}}^{\infty}\frac{{\rm d}r}{r^2}\\ &= \frac{2e^2\mathbf{v}_{\rm o}}{3r_{\rm o}\,c^2}\,. \end{aligned} \tag{11}$$

This is the electromagnetic field momentum in the volume outside the sphere of radius $r_{\rm o}$. One gets exactly the same expression (Eq. (2)) for the electromagnetic field momentum for a charge moving with a uniform velocity equal to the "present velocity", $\mathbf{v}_{\rm o}$, of the uniformly accelerated charge.

It has been shown explicitly elsewhere that the self-field energy-momentum of a charge moving with a uniform velocity can be represented by the kinetic energy-momentum of the charge, provided its electric mass is taken as its inertial mass [29, 31]. It has also been shown that for a uniform acceleration, the contribution of the acceleration fields to the total field energy of the charge is just sufficient to match exactly the amount needed for its velocity-dependent self-field energy based on its extrapolated motion at a future time [34]. This is possible since both make use of the velocity and acceleration of the charge at t', and things in mechanics and electrodynamics are such that the rates of change of energy from both at any later time $t(>t')$ remain synchronized for a uniformly accelerated charge. We have also presently shown that for a uniformly accelerated charge, but with a non-relativistic motion (see [34] for a full relativistic treatment), total Poynting flux, including from both velocity and acceleration field terms, at any time is just equal to that of a charge moving uniformly with a velocity equal to the instantaneous "present" velocity of the accelerated charge. Further, it was shown that there is no excess flux in fields that could be treated as radiation, over and above that implied from the instantaneous "present" velocity of a uniformly accelerated charge.

It follows that in the case of a *uniformly* accelerated charge, its rate of change in kinetic energy is concurrent with the rate of change in its electromagnetic field energy $\mathcal{P}_{\rm field}$, and is therefore given by the scalar product of the rate of change of its electromagnetic field momentum $\dot{\mathbf{P}}_{\rm field}$, with its instantaneous velocity $\mathbf{v}_{\rm o}$.

$$\mathcal{P}_{\rm field} = \dot{\mathbf{P}}_{\rm field}\cdot\mathbf{v}_{\rm o} = \frac{2e^2\dot{\mathbf{v}}\cdot\mathbf{v}_{\rm o}}{3r_{\rm o}\,c^2}\,. \tag{12}$$

4 Radiative losses from a charge moving arbitrarily

As long as the charge continues to move with acceleration *equal to that at the retarded time* (i.e., a uniform acceleration) no mismatch in field energy takes place. However, a mismatch in the field energy with respect to the kinetic energy of the charge could occur when charge moves with a non-uniform acceleration since there is no information in the field expressions about the rate of change of acceleration of the charge (cf. Eq. (3)). In that case the "real" velocity of the charge differs from the extrapolated value obtained from the value of acceleration at the retarded time and the kinetic energy due to the actual velocity no longer agrees with that determined by the acceleration at the retarded time. Then the total energy in electromagnetic fields does not correspond to that expected in self-field because of the "real" velocity of the charge, and it is this difference in the field energy that could be said to be the power loss due to radiation. Thus, in the case of a non-uniform acceleration there will be a mismatch in the field energy with respect to the kinetic energy, calculated from the actual velocity of the charge, since the rate of change of acceleration $\ddot{\mathbf{v}}$ does not enter in the electromagnetic field expression (Eq. (3)), while it does determine the actual velocity of the charge (after all that is how $\ddot{\mathbf{v}}$ gets defined).

We consider a non-relativistic motion of a uniformly charged spherical shell of radius r_o, moving initially with a uniform acceleration $\dot{\mathbf{v}}$ up to some time t' and then a rate of change of acceleration, $\ddot{\mathbf{v}}$, is imposed on the charge motion. Now at a time $t_o = t' + r_o/c$ the information about the change in acceleration has not yet gone beyond r_o, hence the electromagnetic fields and the energy-momentum in them outside the charge radius r_o are unaffected by the imposition of $\ddot{\mathbf{v}}$ on the charge motion at t'. Therefore the electromagnetic energy-momentum in fields external to the charge continues at t_o to be that of a uniformly accelerated charge, and thus determined from $\mathbf{v}$ and $\dot{\mathbf{v}}$ at t'. Thus energy in the fields mimics the extrapolated value of the kinetic energy of the charge, with electric mass of the charge taken as its inertial mass, for its erstwhile uniform acceleration [29, 31, 34]. However, due to a change in the acceleration ($\ddot{\mathbf{v}}$), the actual kinetic energy of the charge at t_o is no longer that determined from $\mathbf{v}$ and $\dot{\mathbf{v}}$ alone, as it will contain $\ddot{\mathbf{v}}$-dependent terms too. Thus by comparing the change in the mechanical power between the two cases (i.e., uniform acceleration and non-uniform acceleration cases), one should be able to calculate the excess power going in the fields above the actual rate of change of the kinetic energy of the charge.

Laws of mechanics determine the actual charge motion at t_o, taking $\ddot{\mathbf{v}}$ also into consideration

$$\mathbf{v}_o = \mathbf{v} + \dot{\mathbf{v}}(t_o - t') + \frac{\ddot{\mathbf{v}}(t_o - t')^2}{2}, \quad (13)$$

$$\dot{\mathbf{v}}_o = \dot{\mathbf{v}} + \ddot{\mathbf{v}}(t_o - t'). \quad (14)$$

The electrodynamics fields (Eq. (3)) do not take into consideration any rate of change of acceleration. For a finite rate of change of acceleration, the velocity and thereby kinetic energy of charge at t_o would contain $\ddot{\mathbf{v}}$, meaning charge would have different kinetic energy than what went into its electromagnetic fields, the latter not taking $\ddot{\mathbf{v}}$ into account.
The expression for the kinetic power is,

$$\mathcal{P} = \mathrm{d}(m_{el}\mathbf{v}_o^2/2)/\mathrm{d}t = m_{el}\dot{\mathbf{v}}_o \cdot \mathbf{v}_o\,, \qquad (15)$$

which for a uniform acceleration case ($\ddot{\mathbf{v}}_o = 0$) from Eqs. (5) and (6) is,

$$\mathcal{P}_1 = m_{el}\dot{\mathbf{v}} \cdot \Big[\mathbf{v} + \dot{\mathbf{v}}(t_o - t')\Big]. \qquad (16)$$

The expression for the power going into the kinetic energy of the charge in a non-uniform acceleration case ($\ddot{\mathbf{v}}_o \neq 0$), from Eqs. (13), (14) and (15) is

$$\begin{aligned}\mathcal{P}_2 &= m_{el}\Big[\dot{\mathbf{v}} + \ddot{\mathbf{v}}(t_o - t')\Big] \\ &\cdot\Big[\mathbf{v} + \dot{\mathbf{v}}(t_o - t') + \frac{\ddot{\mathbf{v}}(t_o - t')^2}{2}\Big]. \qquad (17)\end{aligned}$$

But this is not the power going into the changing electromagnetic fields of the charge, which does not involve $\ddot{\mathbf{v}}$ (see Eq. (3)) and is thus still given by Eq. (12), and equals $\mathcal{P}_1$ (Eq. (16)). The excess power, $\Delta\mathcal{P}$, going into the fields over and above the actual kinetic power of the charge ($\mathcal{P}_1 - \mathcal{P}_2$) then is,

$$\begin{aligned}\Delta\mathcal{P} &= m_{el}\dot{\mathbf{v}} \cdot \Big[\mathbf{v} + \dot{\mathbf{v}}(t_o - t')\Big] \\ &\quad -m_{el}\Big[\dot{\mathbf{v}} + \ddot{\mathbf{v}}(t_o - t')\Big] \\ &\quad \cdot\Big[\mathbf{v} + \dot{\mathbf{v}}(t_o - t') + \tfrac{\ddot{\mathbf{v}}(t_o - t')^2}{2}\Big], \qquad (18)\end{aligned}$$

which to the lowest order in $t_o - t'$ $(= r_o/c)$ is

$$\Delta\mathcal{P} = -m_{el}\ddot{\mathbf{v}} \cdot \mathbf{v}_o(r_o/c). \qquad (19)$$

Substituting for the electric mass of a charge, $m_{el} = 2e^2/3r_oc^2$, the excess power in the electromagnetic fields is,

$$\Delta\mathcal{P} = -\frac{2e^2}{3r_oc^2}\ddot{\mathbf{v}} \cdot \mathbf{v}_o\frac{r_o}{c} = \frac{-2e^2\ddot{\mathbf{v}} \cdot \mathbf{v}_o}{3c^3}. \qquad (20)$$

This is the formula for power losses from a radiating charge.

We can write this power loss being due to a radiative drag force $\mathbf{F}$ as the charge moves with a velocity $\mathbf{v}_o$.

$$\Delta\mathcal{P} = -\mathbf{F} \cdot \mathbf{v}_o = \frac{-2e^2\ddot{\mathbf{v}} \cdot \mathbf{v}_o}{3c^3}. \qquad (21)$$

or

$$\Big[\mathbf{F} - \frac{2e^2\ddot{\mathbf{v}}}{3c^3}\Big] \cdot \mathbf{v}_o = 0. \qquad (22)$$

Since in Eq. (22) $\mathbf{v}_o$ is an arbitrary vector, implying that the equation is true for all values of $\mathbf{v}_o$, we have

$$\mathbf{F} = \frac{2e^2\ddot{\mathbf{v}}}{3c^3}. \qquad (23)$$

Here it could be objected that one could add to $\mathbf{F}$ any arbitrary vector $\mathbf{A}$ such that

$\mathbf{v}_o \times \mathbf{A} = 0$, and still satisfy Eq. (22). For instance, the force on a charge in a magnetic field $\mathbf{B}$ is proportional to $\mathbf{v}_o \times \mathbf{B}$. However, since no power loss results in such a case, it does not represent any radiation reaction force. Therefore Eq. (23) remains valid for the radiation reaction force.

This formula for radiative drag force or radiation reaction is the same as derived in literature from the self-force of a charged sphere. But we have here derived radiation reaction and the radiative losses from the kinetic power of the electric inertial mass of a charged particle.

It has sometimes been stated in the literature [16] that radiation reaction is not represented correctly by Eq. (23), and that it should instead be calculated from Larmor's formula for radiative losses. However, that is an erroneous statement and by using synchrotron radiative losses as an example, it can be conclusively demonstrated that the radiation damping calculated from Larmor's formula (or its relativistic generalization Liénard's formula) does not yield results compatible with the special relativity and further, violates energy-momentum conservation [26].

In an assumedly uniform and homogeneous magnetic field, a charge will be moving in a helical path with a velocity component $\mathbf{v}_\| = \mathbf{v}\cos\psi$, parallel to the magnetic field, where ψ is the pitch angle (i.e., angle with respect to the magnetic field vector) of the charge. Since the radiation is confined to a narrow cone around the instantaneous direction of motion of the charge [5, 8, 35], from Larmor's formula (or rather from Liénard's formula), any radiation reaction on the charge will be in a direction just opposite to its instantaneous velocity vector [36, 37], implying no change in the pitch angle of the charge. Thus the ratio $\mathbf{v}_\perp/\mathbf{v}_\| = \tan\psi$, will not change. However, there is something amiss in the above arguments and the above picture is not consistent with the special theory of relativity. A more careful consideration shows that in the case of synchrotron losses, the velocity component parallel to the magnetic field ($\mathbf{v}_\|$) of the charge remains unaffected, while magnitude of $\mathbf{v}_\perp$ steadily decreases due to radiative losses and as a consequence the pitch angle of the radiating charge in general changes, with the charge motion gradually getting aligned with the magnetic field direction [27]. Thus the dynamics of the charged particle computed from Larmor's formula (or its relativistic generalization Liénard's formula) does not yield results compatible with the special relativity and that only the radiation reaction formula yields a picture consistent with the special relativity.

5 Conclusions

From mechanical considerations of electric inertial mass of a charge, formulas for radiation reaction and radiative losses were derived, albeit in a heuristic manner. The derivation made use of the fact that,

1. A moving charge has an electromagnetic field momentum which we can infer from classical mechanics if one uses its electric inertial mass.

2. In the case of a uniformly accelerated charge, its rate of change in kinetic energy is concurrent with the rate of change in its electromagnetic field energy, and is given by the scalar product of its instantaneous velocity with the rate of change of its electromagnetic momentum.

3. In the case of a varying acceleration, the energy in the electromagnetic fields changes at a different rate than that of change of kinetic energy of the charge and it is this energy difference that is not represented in the actual motion of the charge and can be called as a radiative loss.

The accordingly derived instantaneous power loss turns out to be directly proportional to the scalar product of the velocity and the rate of change of acceleration of the charge as derived earlier in literature from radiation reaction due to the self-force of the charge.

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Teaching Methodology for the Introduction of Quantum Mechanics

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Abstract

In most of the universities and colleges, introduction of Quantum Mechanics starts with failures of Classical Mechanics. Mathematics needed to understand quantum mechanics (operator formalism etc.) starts quite abruptly after that. This paper discusses a simple teaching approach to introduced Quantum Mechanics to the newcomers by answering some outstanding questions which students are hesitant to ask.

1. Introduction

Generally, introduction of quantum mechanics starts with failures of Classical Mechanics. Assumptions which are introduced to solve these failures are said to be part of Quantum Mechanics. Wave function and its physical significance, Schrödinger equation and a few problems which can be exactly solved are then discussed. Postulates of operator formalism of Quantum Mechanics and its mathematics start after that. This is the approach taken by most of the UG/PG teachers of the University. Although there is nothing wrong in this approach. But more often, students are left with many unanswered questions. Some of the questions do not arise because of the approach and some questions which arise in their mind get suppressed because of the totally new view of looking into systems as compared to Classical Mechanics.

Some of the outstanding questions students faces are;

- Whether Quantum Mechanics is a totally new subject/concept
- What is the need of wavefunction and why it has to be a complex function
- Where exactly the transition lies between Quantum world to Classical world
- Why there is a Uncertainty relation and whether it is possible to experimentally verify it
- Why there is any need of Vector space, Hilbert space, operators which are linear and Hermitian in nature

This paper discusses a teaching approach to understand the answers of all these above mentioned questions which often students hesitate to ask and also are unable to find them in standard textbooks [1, 2].

2. Introduction to Quantum Mechanics

It is a wrong notion which most of the students carry that Quantum Mechanics is the approximation of Classical Mechanics. It is mainly because they see probabilities and approximation everywhere in Quantum Mechanics. In fact, Classical Mechanics is the approximation of Quantum Mechanics [3]. One can derive the Newton's laws of motion which form the basis of Classical Mechanics from Schrodinger's equation. But the Schrodinger's equation itself is non derivable and stands fundamental principle in itself. In many books, it is derived using a free particle example which leads to this misunderstanding. Hence students assume that it is derivable but as a matter of fact it is not [3]. Quantum Mechanics is not a totally new subject or concept. Rather it is mixture of subject like optics, mathematics, and statistics. Relation between Quantum Mechanics and optics lies in the wave-particle duality. Much experimental verification had done to verify both light as well as particle behaves as both particle and wave like. This leads to concept of wave packet. Wave packet is neither a wave nor a particle. It is hanging in between wave and particle. But is has all the essential properties of wave and particle. Many derivations and concepts in optics are taken as it is in Quantum Mechanics. Mathematics and statistics needed for Quantum Mechanics is also not new [4-5]. It was known before. This point should be emphasis to students so that the fear factor of learning entirely new view or concept will go away. Introduction to nine formulations of Quantum Mechanics [6] to the students is also an essential step in the first year Quantum Mechanical course. Mathematical analogies and differences between Quantum Mechanics and Classical Mechanics should be properly conveyed to the students. For example, Quantum Hamiltonian-Jacobi formalism and it's analogy in Classical Mechanics [7].

3. Complex wavefunction

Wavefunction has to be complete description of the quantum system then it has to be a complex function. Complex numbers were invented because of our inability to tell the root of -1. This inability leads us to call it an imaginary number denoted by i. However if we square it up then it yields an exact answer, a real number that is -1. If the wave function is complex, then it will behave in the same way. But to know why it has to be a complex function consider following arguments. Due to wave-particle duality, one needs to consider concept of wavepacket. The most generalized solution of a wave equation is given by,

$$A(z,t) = Ce^{\pm i[(kz \mp \omega t]} \qquad (1)$$

Eq. (1) shows that A(z,t) is a complex function. Similarly if the wave function has to be a complete description of the system, then it has to be complex function.

4. Transition between Classical and Quantum Mechanics

Often, students read in the textbook that classical mechanics is applicable to macroscopic world and quantum mechanics is applicable to microscopic world. However, it is difficult to pinpoint at which size scale exactly the transition between Quantum Mechanics and Classical Mechanics happens but surely it is related to magnitude of Planck's constant. As we see below, magnitude of Planck constant 'h' plays very important role in defining the boundaries of "classical world" and "quantum world". Consider the magnitude of h, its value is 6.634×10^{-34} J.s, which is ridiculously small number and in good approximation equivalent to ZERO. Then why it is so important in Physics? Importance of Planck's constant comes from its ability to differentiate between Quantum and Classical world. Any change in the Planck constant value directly affects the location of border/boundary between the Quantum and Classical world. As of today we understand that to observe Quantum effects we need to look into atomic and molecular world i.e. Microscopic world and classical effects are connected with our day to day experiences i.e. Macroscopic world. In other words as we increase the size of the objects then one can see the transition between Quantum world to classical world. Only thing is we don't know yet what the exact size is, where this transition happens.

Let us think of two cases where Planck's constant 'h' is different from its value of 6.634×10^{-34} J.s. Imagine there exists a switch with which we can change the value of 'h' from its current value. However there is no guarantee that Universe will remain same and life forms will survive this change. We will consider following two cases based upon this.

Case I: When h=6.634×10^{-33} J.s

Suppose the magnitude of h is one order more than the current value. De Broglie wavelength is defined by $\lambda=h/p$ where p is the momentum. So comparable to existing world, wavelength in this world will be one order of magnitude greater. If the wavelength becomes large then it will easier to observe the quantum effect. So size wise we will see the transition between Classical to Quantum World bit early than the current one.

Case II: When h=6.634×10^{-35} J.s

In this case 'h' is smaller than an order of magnitude compared to current one. De Broglie wavelength becomes smaller in this world. Hence if the wavelength becomes small then it will become difficult to observe the quantum effect. So size wise we will see the transition between Classical to Quantum World bit later than the current one.

5. Uncertainty relation

Wave particle duality and wavepacket concept in Quantum Mechanics are responsible for Heisenberg uncertainty principle. The entire derivation of uncertainty principle from the definition of uncertainty in statistics is very well given in almost all the leading text books of Quantum Mechanics. Researchers nowadays are investigating rigorous interpretation of Heisenberg's uncertainty statements [8, 9].

In the normal text books, uncertainty relation is further explained using thought experiment. Student often assume that the experimental realization of uncertainty relation is very difficult and hence has not done till now. So that is why a thought experiment is given in most of the books. Given below a simple explanation of energy time uncertainty equation

Consider the energy time uncertainty equation and let us assume that it is approximately equal to Planck constant 'h' (although it is $\hbar/2$).

$$\left.\begin{aligned} &\Delta E.\Delta t \sim h \\ &\because \Delta E = h\Delta\nu \\ &\Rightarrow h\Delta\nu\,\Delta t \sim h \\ &\Rightarrow \Delta\nu.\Delta t \sim 1 \end{aligned}\right] \qquad (2)$$

Now let us analyze the last relation i.e. $\Delta\nu.\Delta t=1$, by considering the case of continues wave (cw) laser and pulsed laser.

Case I

Let us consider the case of cw laser first and recall the basic properties of a LASER. Laser light is highly monochromatic, coherent and directional. Monochromatic means it emit only one frequency, so if we keep a spectrometer in front of it, we will be able to see that the frequency vs intensity graph shoots up only at one particular frequency (with ideally zero width) and then it goes to zero as shown in following fig. 1. Fig. 1 resembles to graph of delta function.

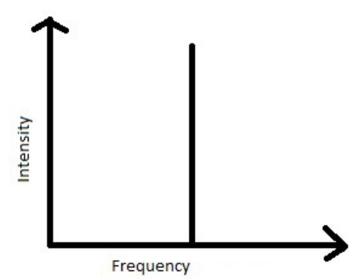

Fig 1: Frequency vs intensity graph of monochromatic continuous wave laser.

Let us now write the frequency time uncertainty relation once again

$$\Delta\nu.\Delta t = 1$$

Now since the light is continuously coming out of the laser system, Δt in this case is infinite. So using above relation

$$\Delta\nu = 1/\infty = 0$$

This implies that the frequency width is zero or in other word cw laser is monochromatic in nature. Also we know that Fourier transform of delta function is a constant function.

Case-II

Now consider the case of pulsed laser, light from this laser is not coming out continuously, it is coming in the form of pulses having finite duration ranging from few μs (10^{-6} sec) to attoseconds (10^{-18} sec). In this case Δt is finite but very small. According to uncertainty relation if Δt is finite but very small, then Δν should be also finite. This implies that pulsed laser is not monochromatic, it is polychromatic. In reality, pulsed laser is having frequency width as predicted by the uncertainty relation and can be easily experimentally verified.Hence this uncertainty principle can be verified experimentally in the laboratory having cw and pulse laser.

6. Concept of Hilbert Space and Operators

Let us assume a two level atomic system as shown in Fig.2. Wave function of ground level is taken to be ψ_1 and that of the excited level denoted by ψ_2. Let the energy of ground and excited state be represented by E_1 and E_2.

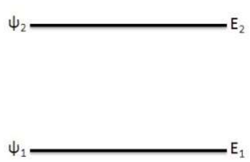

Fig 2: Two level atomic system

Assume E_1 and E_2 are experimental values of energy of level ψ_1 and ψ_2. Ideally E_1 and E_2 should be equal to energy eigenvalues, if the exact form of ψ_1 and ψ_2 is known. So the best way to verify a quantum theory is to match calculated eigenvalues with the experimental results if the eigenfunctions are known. This point should be emphasis to students, because this is the link between theoretical concepts and experimental results. To match the calculated eigenvalue with the experimental result (a real number), eigenvalue should be a real number. Only Hermitian operators can yield a real eigenvalue. That is why operators in Quantum Mechanics are Hermitian operators. According to postulates of operator formalism, any arbitrary wavefunction ψ representing a quantum system such as a two level atomic system can be given by

$$\psi = C_1\psi_1 + C_2\psi_2 = \sum_{n=1}^{2} C_n\psi_n \qquad (3)$$

Where, $|C_n|^2$ equals the probability that the system is in the state ψ_n. There are infinite numbers of values C_n can take. Hence number of ψ's which represent a two level quantum system are infinite. So even if we consider a simplified two level system, number of ψ's which can represent the system are infinite. In Quantum Mechanics, we represent all these ψ's on Hilbert space. Hilbert space is defined as "A complete and infinite-dimensional complex Euclidean Space". In a realistic quantum system, there are infinite numbers of levels. So the number of eigenfunctions (and hence basis) will become infinite. Hence the Vector space becomes infinite dimensional. Numbers of arbitrary vectors which also represent the system grow even larger because of the inclusion of infinite number of basis vectors. In fact, number of arbitrary vectors grew so large that each Cauchy sequence in a vector space having n-dimension denoted by H_n converges to a limit in H_n ($||\psi_i-\psi_k|| \rightarrow 0$, for i and k $\rightarrow\infty$). This is the exact requirement for a Vector space to be complete. Wave function is a complex function; therefore we need to consider a complex Euclidean space. This is the reason, why we need to define wavevectors and operators on Hilbert space. Once this point is clear to students, rest is all mathematics.

7. Conclusion

A simplified teaching methodology is presented for the benefit of students. Although the complete introduction to the subject is not presented, nevertheless it provides answers to some outstanding questions which students face while dealing with the Quantum Mechanics for the first time. Classroom feedback to this introduction was very well appreciated by students. Many of them were surprised by this unorthodox approach to the beginning of their journey in Quantum Mechanics.

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