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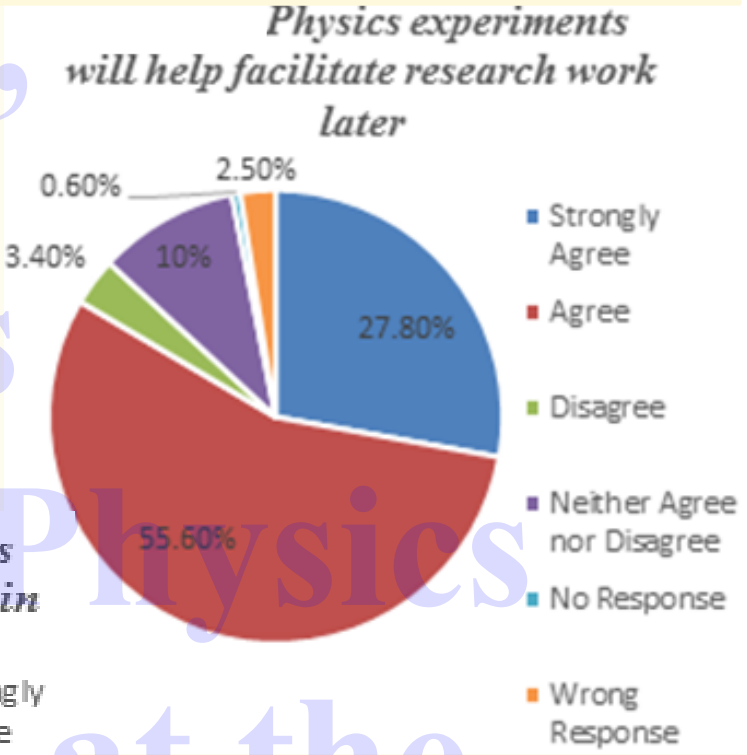
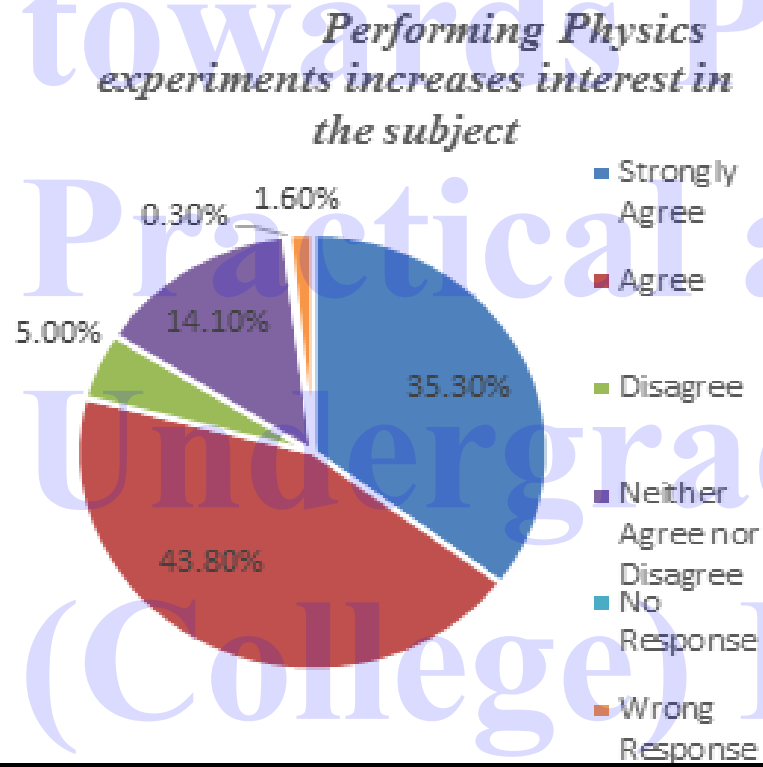
Students' Attitudes towards Physics Practical at the Undergraduate (College) Level





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In this Issue

EDITORIAL

As this issue of *Physics Education* reaches the readers, we are still in the midst of the great Indian annual ritual, the undergraduate admission process in the country, with all its highs and lows.

This is too important a subject to be dismissed in a short editorial. There are a variety of entrance exams for various institutions; some like the joint entrance exam (JEE Mains) for admission to IITs/NITs/other centrally funded technical institutions are taken by nearly 13 lakh students. The common entrance exam for undergraduate medical studies NEET is also taken by a similar number. Other aptitude tests such as by IISERs are selective in who are eligible to write the exam in the first place and hence relatively smaller number of students take the exam. By any yardstick, the competition is intense to the point that failure in exams often leads to frustration among students and in some extreme cases has been a cause for suicide as well.

Where does physics as a subject stands in these scheme of things? It is heartening to note that many students compete in entrance exams to study physics at IISc, IITs, IISERs, Hyderabad central university and other central universities. From the informal conversations, it appears correct to believe that some of the best undergraduates do choose to study physics. This is apart from the large numbers who have talent and aptitude for physics and are admitted to physics courses in colleges and universities based on the marks in their qualifying exam but without any screening test. Clearly, across the country, there is no dearth of incoming talent in physics.

Inspite of what appears to be a rosy situation at least for physics, when we strike a conversation with seasoned and veteran teachers in institutions ranging from IITs to universities and colleges, often there is an expression of unease. Many blame the entrance exam effect because while it prepares students to answer multiple choice questions often employing shortcuts, it does not prepare them for tackling challenges that physics seeks to solve in academic research and real-life. Deeper understanding of the subject, physics or any other discipline, takes a back seat. Often, given that the entrance exam preparations are intense, students find it difficult to get over its hang-over even long after the entrance exams are over.

With a growing population and intense competition for quality education, entrance exams has become part of process to the extent that very little thought is given to possible alternatives to this system. Even though it might be a long haul, it is still worthwhile to think about improvements to entrance exam or completely new alternatives to the current pattern of entrance exams for undergraduate admissions.

M. S. Santhanam
Chief Editor
Physics Education

Ensemble Formalism for a Market

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Abstract

In this paper we studied about a simple market using statistical ensembles and obtained the equation of state for a market in micro canonical, canonical and grand canonical formalism.

1 Introduction

Econophysics is a new interdisciplinary subject which use theories in physics to solve problems in economics. The name 'ECONOPHYSICS' was first introduced by H. E. Stanley in Kolkata Conference, 1995 [1]. Econophysics uses the tools of physics like statistical physics, condensed matter physics, etc., in the description of financial systems. Sudden changes in economic policies affect people positively as well as negatively. For example demonetization and now the Goods and Services Tax (GST) introduced in India are directly affecting people. So public is more conscious about economic activities happening in the world, which demands an error free scientific approach to economics. Several analogies between physical and financial systems have been recognized and explored [2]- [4]. Many successful theories are there in Econophysics related to stock markets, growth of companies, speculative markets etc. [2]- [14]. Lots of books are also available in this new subject [15]- [21]. Financial markets are well defined complex systems, which are continuously monitored. Every economic transaction is recorded, and the recorded economic data are accessible to interested researchers which make financial markets extremely attractive for researchers [15]. So we here attempt to introduce the subject Econophysics for students of physics through the basic concepts of statistical mechanics. In our article we use statistical ensemble theory to describe a market and derive the

equation of state [EOS] for a market equivalent to ideal gas equation.

2 Defining market as an ensemble

We consider a simple market where there are N agents (or shops). dQ is the amount of asset put into the market in the form of money m, goods V, labour etc. This input will give rise to increase in the average money per agent T, marketing of goods with price per unit P and other transactions which we will not consider here. Mathematically we can say

$$dQ = dm + dW$$

Taking $dW = PdV$ we get

$$dQ = dm + PdV$$

In thermodynamics for an ideal gas, particles moving with kinetic energy (here money m) colliding with walls of the container and producing an average energy (here average money M which is a function of temperature (here average money per agents) and exerting pressure on the walls of the container(here price of goods /unit). Thus the market we described here is analogous to an ideal gas. So we anticipate there is equation of state [EOS] like

$$P = \frac{NkT}{V}$$

where P is pressure, N the number of particles, k is the Boltzmann constant and V is the volume, for the market. Since there is close analogy we use statistical mechanics concepts to the market, to find the EOS. The positive value of dV represents inward flow of products from the outside of the market which reduces the market capital. The payment for the goods decreases the amount of money. The value of money m circulating on the market is connected with the number of trading units N, the volume of goods V, the prices per unit P. That is, a simple market parameters can be compared with the parameters of an ideal gas. A comparison table is given below.

Ideal gas	**Market**
Energy	Money
T	Average money per agent
P	Price per unit goods
N	Number of agents
S	Capital distribution
A	Helmholtz free money
μ	Market fee potential
ϕ	Landau free money

3 Microcanonical ensemble [MCE] for market

Similar to thermodynamics we can define entropy, a quantity closely connected to the capital distribution in an economic system. In economics work is equivalent to increasing and collecting capital. Collecting (goods, money) is equivalent to reducing entropy. Distribution (of goods, money) is equivalent to increasing entropy. Buying means collecting goods and distributing money. Selling means distributing goods and collecting

money. So

$$dQ = TdS$$

Substituting for dQ we get a relation analogous to the first law in thermodynamics

$$TdS = dm + PdV$$

which may be called as the **First Law of Market**. In microcanonical ensemble we will consider states which are function of m,V and N. So consider two systems with number of ways in which goods are marketed called micro states $\Omega_1(m, V, N)$ and $\Omega_2(m, V, N)$ with entropies S_1 and S_2. If they are allowed to interact, their resultant entropy

$$S = S_1 + S_2$$

It is easy to show that

$$\Omega = \Omega_1\Omega_2$$

As in statistical mechanics we assume $S \propto f(\Omega)$, $S_1 \propto f(\Omega_1)$, $S_2 \propto f(\Omega_2)$. So we get

$$S \propto \ln \Omega$$

Let the constant of proportionality is k and hence the equation takes the form

$$S = k \ln \Omega$$

Let us take k as unity here. This is the bridging equation for MCE in econophysics. From S using Law of market we get

$$\left(\frac{\partial S}{\partial m}\right)_{V,N} = \frac{1}{T}$$

$$\left(\frac{\partial S}{\partial V}\right)_{m,N} = \frac{P}{T}$$

Thus if we can find Ω of any system, we can find out all the parameters of the system from S. Our aim is to derive an equation for price/unit(P) of the market. For n goods to be distributed among N agents is

$$\Omega(m, N) = \frac{(n + N - 1)!}{n!(N - 1)!}$$

For $n \gg N$ using Stirling approximation

$$S = N\left(\ln \frac{N + n}{N} + n \ln \frac{n + N}{n}\right)$$

$$S = N \ln \left(\frac{N + n}{n}\right)$$

$$\frac{P}{T} = \frac{\partial}{\partial V}\left(\ln \frac{n}{N}\right)$$

Then

$$\frac{P}{T} = \frac{1}{V_0}\frac{N}{n} = \frac{N}{V}$$

where $V = nV_0$ and V_0 is the volume of a single object marketed. Therefore equation of state for free market is obtained as

$$P = \frac{NT}{V}$$

analogous to that of ideal gas. This is the relation connecting price, goods flow and average money per agent for any simple market.

4 Canonical ensemble [CE] for market

Next we consider a market system where the number of trading agents N is fixed, but the total asset or money m is not fixed. This way of treating market is called CE formalism. In canonical ensemble, money m of

the system essentially varies but the average amount of money per agent, T is kept constant. To keep T constant the economic system is in contact with some external agency like a reservoir(which can be a bank or any financial institution). Let p_i be the probability of the subsystem being in state i and it is equal to the fraction of the total number of states (of system plus reservoir) in which the subsystem is in the state i. If Ω represents the number of micro states,

$$p_i = \frac{\Omega_{res}(m_{tot} - m_i)}{\Omega_{tot}(m_{tot})}$$

We have $m_{tot} = m_{res} + m_i$ where m_i is the system money at any instant i or state i and Ω_{tot} is the total number of states of the system plus reservoir. m_i is continuously changing. Hence another property of the system which is known to us is introduced which is the average money M, defined as $M = \sum_i p_i m_i$. Adding and subtracting M and taking

$$\Omega = e^S$$

So

$$p_i = \frac{e^{S_{res}(m_{tot} - M - (m_i - M))}}{e^{S_{tot}(m_{tot} - M + M)}}$$

Using Taylor expansion for numerator we get

$$p_i = e^{\frac{-m_i}{T}} e^{\frac{M - TS}{T}}$$

Similar to Helmholtz free energy in thermodynamics, using Legendre transformation Helmholtz free money A is defined as

$$A = M - TS$$

Substituting

$$p_i = e^{-\beta m_i} e^{\frac{A}{T}}$$

where $\frac{1}{T} = \beta$ Since $\sum_i p_i = 1$ and defining $\sum_i e^{-\beta m_i} = Q$, called partition function, we get

$$e^{-\beta A} = Q$$

$$A = -T \ln Q$$

Differentiating and using the Law of Market we get

$$P = -\left(\frac{\partial A}{\partial V}\right)_{N,T}$$

Prices per unit P can be found out.

4.1 EOS

Let the single partition function is

$$Q_1 = \sum_i e^{\frac{m_i}{T}}$$

Taking the phase space of volume and money

$$Q_N(m, V, T) = \int \int d^{3N}v \prod_i d^{3N} m_i e^{-m_i/T}$$

$$= (VT)^N$$

Then Helmholtz free money is

$$A = -T \ln Q_N = -NT \ln(VN)$$

from which marginal price P is

$$P = \left(-\frac{\partial A}{\partial V}\right)_{T,N} = \frac{NT}{V}$$

Using different equations for money for different systems like debt system, credit market, etc., we can find equation of state as well as other quantities like entropy, economic temperature, etc.

5 Grand canonical ensemble [GCE] for market

Now we consider grand canonical ensemble, there neither m nor N is fixed, but average money per agent(T) and the market fee potential (μ) which controls the number of trading units or agents are fixed. In GCE we can show that

$$\phi = -T \ln \mathcal{Z}$$

where $\mathcal{Z}$ is the grand canonical partition function. For the market $\mathcal{Z}$ is

$$\mathcal{Z} = \sum_i e^{-(m_i - \mu N_i)/T} \tag{1}$$

where μ is the market fee potential which controls the number of agents in the market. Here m_i the money with agents and N_i the number of agents vary. Taking $z = e^{\mu/T}$ and substituting we get,

$$\mathcal{Z} = \sum_{N_i} z^{N_i} Q_{N_i} = \sum_{N_i} (VTz)^{N_i} = \frac{1}{1 - zVT}$$

So

$$\ln \mathcal{Z} = -\ln(1 - VTz)$$

$$\phi = T \ln(1 - VTz)$$

5.1 EOS

Marginal price P is,

$$P = -\left(\frac{\partial \phi}{\partial V}\right)_{T,\mu} = \frac{zT^2}{1 - zVT}$$

Number of agents

$$N = \left(\frac{\partial \phi}{\partial \mu}\right)_{V,T} = \frac{zVT}{1 - zVT}$$

Substituting N

$$P = \frac{N\,T}{V}$$

Thus here also we obtain the same EOS.

6 Market Fluctuations

Many complex systems exhibit sudden changes in behaviour or structure which is called phase transition. This can be applied to dynamics of financial markets also. For some economic systems a phase transition may be a change in the macroeconomic structure, like a change from a raw material economy to a high technology economy, or it may be the introduction of a reduced bank rate. Sudden macroscopic changes in the character of financial system is known as market crash. The transition is called first order or non equilibrium if they occur spontaneously and suddenly in response to external conditions and once started it can not be controlled. In such cases even if the external causes are removed the system continues in the same state for some time. The second order transitions are equilibrium transitions where the change is gradual [22]. In many types of transitions the changes are associated with correlation of fluctuations of the system parameters. In markets the growth and decay of correlation is associated with changes in the market conditions, such as correlation between the price of petroleum products and other commodities. In physics we define free energy A which is the 'work' associated with creating

a given state of the system. Similar to free energy we can define free flow of money A for a market. In financial systems economic temperature of a country is given by mean GDP per household or standard of living and when T reaches critical value there is a sudden change in the value of free flow of money which leads to market crash. Analysis of free flow of money helps in forecasting market crashes. The Landau theory explains how such a phase transition in physical systems can be expressed in terms of an order parameter thus describing the general properties of the system without examining their microscopic properties. Similarly macro mechanisms of market transitions can be explained without knowing the individual issues. We can apply Landau theory of phase transitions [23] for detailed analysis of market transitions which is beyond the scope of this article.

Conclusion

In this article we identified the equivalence of thermodynamical quantities with economic transactions and derived a law of the market equivalent to first law of thermodynamics. Then we used statistical thermodynamics to study different economic systems. We derived the equation of state of a market using the three ensembles which is similar to ideal gas equation. This article also shows the role of free flow of money in forecasting market crashes by using the analogy with free energy. In short we emphasize the role of statistical physics tools in the study of economics.

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Loop-the-loop Demonstration with Rolling Radius Correction

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Abstract

In this article we present a careful analysis of the kinematics and dynamics of a rolling object on a concave curved surface, in particular the loop-the-loop track. It is shown that modeling this rolling motion as a superposition of translational motion and pure rotation with respect to the center of mass, often used in some studies, is incorrect for an object rolling on a *curved* surface. We present a revision of the kinematic equations and kinetic energy of a rolling body with respect to a laboratory frame that accounts for both the rolling motion of the object and the circular motion of the object moving along a curved surface. In solving the loop-the-loop problem of determining the minimum height to release a rolling object to travel around the inside of the loop without losing contact, our analysis presents a solution that includes a correction term, absent from commonly presented solutions, that includes the radius of the rolling object.

Introduction

Loop-the-loop motion is described in many introductory physics textbooks as a validation of the law of mechanical energy conservation and exhibition of centripetal force. The problem of a block sliding down a frictionless surface, which makes a loop-the-loop in a circle of radius *R*, can be found in almost every introductory textbook. The similar problem and lecture demonstration of ball rolling [1] along the loop-the-loop is an effective verification of essential physics concepts. This paper is devoted to a correction in the calculation of the minimum height *H*, relative to the bottom of the loop, where a solid sphere should be placed and started from rest in order that it may make the loop while remaining in contact with the surface. Additionally, an analysis of typical errors for kinematics and dynamics of rolling object on a concave surface is considered.

Introductory physics textbooks consider a ball and a track as rigid objects (rolling friction is neglected [2]) and assuming that the ball is rolling without sliding (Fig. 1, parameters *H, r, R* and forces *mg* and *N* (normal) are obvious.). Although the assumption of absence of sliding can be violated for some specific conditions [3], this assumption is accepted in this research.

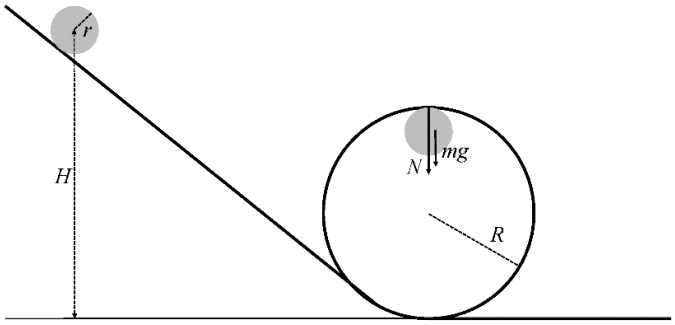

Fig. 1 Solid sphere on a loop-the-loop.

To find the minimal *H* for which the solid sphere can make the loop, the law of energy conservation and the theorem of motion of the center of mass should be used. Let the initial ball's speed equal zero. Then by choosing the initial position and the position of the ball at the highest point of the loop as two states, the equation of mechanical energy conservation can be written as

$$mgH=mg\,(2R-r)+K\,, \qquad (1)$$

where K is the kinetic energy of the ball. Usually introductory physics textbooks present kinetic energy of the rigid object in the form

$$K=\frac{mv_{cm}^2}{2}+K_r, \qquad (2)$$

where v_{cm} is the speed of the center of mass, $K_r = \frac{I_{cm}\omega_1^2}{2}$ is kinetic energy of rotational motion with respect to the axis coming through the center of mass and ω_1 is the angular speed of the ball. Eq (2) is the so-called König's theorem [4,5] which states that kinetic energy of the system is equal to the sum of kinetic energy due to a particle having a mass equal to the total mass of the system and moving with the velocity of the center of mass and kinetic energy due to the motion of the system particles relative to its center of the mass. This is the kinetic energy of the system as viewed by an observer where the center of mass is translating but not rotating. It means that the kinetic energy of a rolling rigid object is the kinetic energy of translational motion associated with the movement of the center of mass and the kinetic energy associated with the movement of the particles relative to the center of mass (rotational motion). In other words, it is important that König's theorem can be applied in the form of Eq (2) only if the frame of coordinate associated with the center of mass is in translational motion with respect to the laboratory frame.

Kinetic Energy of Rolling Object on a Concave Surface, e.g. Loop-the-Loop Revolution

The necessary condition mentioned above for using Eq (2) is often ignored. Eq (2) is used in almost all introductory physics textbooks for solving the problems of rolling along the plane surface. Indeed, this is a legitimate approach as in this case the motion of the object can be considered as superposition of translational and rotational motions. On the contrary, in the presented case the center of mass of the ball is moving along a circular path, and the system of coordinates associated with the motion of the center of the mass is rotating with respect to an inertial frame placed at the center of the loop (Fig. 1). This is a rotational motion and not translational, therefore König's theorem must be presented in a different form[7], but not by Eq (2). Actually, the kinetic energy of the ball is calculated purely from the kinetic energy of rotational motion about an axis at the instantaneous point of contact, O. The rotation at point O is viewed as superposition of the separate rotational motions of the ball about the loop center and the ball's own center of mass. The ball is rotating with angular velocity ω_2 with respect to the inertial (laboratory) frame in the loop center at the point O_2 and with angular velocity ω_1 with respect to the ball center of mass at point O_1. As a result of the addition of these two angular velocities, the instantaneous velocity at the point O is equal to zero and the ball rolls without sliding on the loop surface. Therefore, the speeds due to rotations with respect to the points O_1 and O_2 are equal (Fig. 2)

$$v_1 = v_2, \omega_1 r = \omega_2 R$$

and therefore, instantaneous angular speed with respect to point O can be found $\omega = \omega_1 - \omega_2 = \omega_1\left(1-\frac{r}{R}\right)$. A detailed analysis of the kinematics of rolling without sliding on a circular surface is given in Ref. 8.

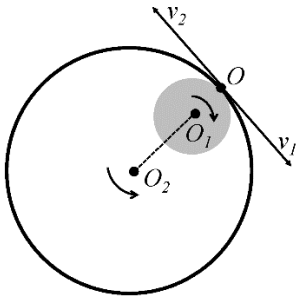

Fig. 2. Description of the rolling of the ball as superposition of the rotation with respect to point O_2 with angular speed ω_2 and rotation with respect to the point O_1 with the angular speed ω_1. Instantaneous velocity of the rolling ball at the point O is equal to zero and angular speed with respect to this point is $\omega = \omega_1 - \omega_2$.

Now kinetic energy of the ball, K, with respect to the laboratory frame (O_2 point), can be calculated because knowing the angular speed with respect to instantaneous O-axis allows us to analyze the ball as

a rigid object in pure rotation about this axis. Kinetic energy of rotational motion is

$$K=\frac{I\omega^2}{2}=\frac{(I_{cm}+mr^2)\omega_1^2(R-r)^2}{2R^2}, \quad (3)$$

where I is the moment of inertia with respect to O-axis and I_{cm} is the moment of inertia with respect to O_1-axis. As $v_{cm}=(R-r)\omega_2=\frac{r(R-r)}{R}\omega_1$, the kinetic energy can be written as

$$K=\frac{mv_{cm}^2}{2}+\frac{I_{cm}\omega_1^2}{2}\frac{(R-r)^2}{R^2}=\frac{mv_{cm}^2}{2}+K_r\frac{(R-r)^2}{R^2}. \quad (4)$$

One can see that Eq (4) is different from usually used Eq (2). However, for $R>>r$, $\omega_1=\omega$ (this is the case for rolling along a plane surface, which is presented in almost all physics introductory textbooks) and Eq (4) converts to Eq (2). Substituting K from Eq (3) to the Eq (1) leads to

$$mgH=mg(2R-r)+\frac{I\omega^2}{2}. \quad (5)$$

Using the moment of inertia of a solid sphere with respect to O-axis, $I=\frac{7}{5}mr^2$, and $v_{cm}=\omega r$
Eq (5) can be written as

$$mgH=mg(2R-r)+\frac{7}{10}mv_{cm}^2. \quad (6)$$

At the highest point of the loop, the net force applied to the ball is directed to the center (centripetal force, F_c) and according to the theorem of the motion of the center mass one can write

$$F_c=\frac{mv_{cm}^2}{R-r}=mg+N.$$

For the critical case of making the loop, $N=0$ and therefore $v_{cm}^2=g(R-r)$. Substituting this v_{cm}^2 in Eq (6) defines the minimum H to make the loop-the-loop

$$H=2.70R-1.70r. \quad (7)$$

Discussion

For the simple case of frictionless sliding of a small block, there is no rotational motion (K_r is equal zero in Eq (1)) and the minimum is $H_1=2.50R$ (calculations can be easily made). For rolling a small ball ($r<<R$), when kinetic energy can be approximated by Eq (2), $H_2=2.70R$. H_2 does not depend on ball radius. The "correction" from taking into account the rotational motion of the ball leads to increasing H (about 8% compared to a block sliding). Taking into account the radius of the ball leads to Eq (7), which indicates that H does depend on the ball's radius and $H<2.70R$. Using a ball with $r=2.50$ cm for Cenco's loop apparatus [1] ($R=22.0$ cm) gives $H_3=2.51R$, which is about 8% less than the $H=2.70R$ result from calculations based on the incorrect use of Eq (2), that ignores the condition that the loop path is circular and not a flat plane. This consideration of the kinetic energy of the ball as translational kinetic energy of the center of mass and its rotation with respect to the center of mass is correct for the motion along a plane (but not circular) path. For this case of r and R values, it should be noted that ($H_2-H_1\approx H_2-H_3$, meaning the "correction" to the height calculation because ball is rolling and not sliding is approximately the same size as the "correction" we present in this paper because the kinetic energy of a ball rolling in a circular loop cannot be correctly calculated from methods that apply to rolling on a flat plane. It means that for use of the existing apparatus for loop-the-loop demonstration, presented in this paper, correct calculations of kinetic energy must be made to accurately determine H under the assumptions made.

Additionally, to demonstrate that H does depend on the ball radius and Eq (7) must be used for H calculation, let us consider the case for $r=\frac{R}{2}$. According to Eq (7) $H=1.85R$ in this case, i.e. H is considerably less than $2.70R$ and actually less than $2.00R$ (Fig. 3). It means that the ball can be released at H significantly less than loop diameter to make a loop-the-loop circle.

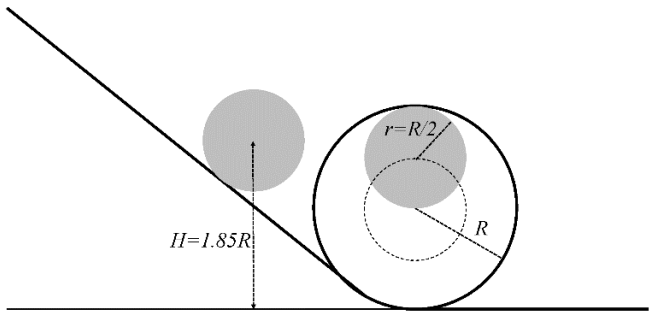

Fig. 3. The initial position and the position of the ball at the highest point for *2r=R*. Dashed line is a trajectory of the center of mass.

It should be mentioned that Eq (7) is also obtained in Ref. (9,10) by using two wrong assertions: for the kinetic energy

$$K=\frac{mv_{cm}^2}{2}+\frac{I_{cm}\omega_1^2}{2} \quad (8)$$

and the kinematic relation between the speed of the center of mass and the angular speed of the ball with respect to the center of mass

$$v_{cm}=\omega_1 r. \quad (9)$$

Eqs (8), (9) are applicable to a ball rolling on a flat surface without sliding, but not to the case considered. König's theorem (Eq (2)) can be applied for a rigid object if its motion can be presented as a superposition of translational and rotational motions. However, in the examined case there is no translational motion but there are two rotations with respect to the O_1 and O_2 axes in different frames, as O_1 axis is rotating with respect to O_2 axis. It means that the problem of vector addition of angular velocities ω_1 and ω_2 must be solved to find the resultant of angular velocity in the laboratory system and the position of the instantaneous axis that defines speed and kinetic energy in the laboratory system, which is presented in this research. The analysis presented in this paper, which is applicable for any $r<R$, shows that kinetic energy is given by
Eq (3) and

$$v_{cm}=\frac{r(R-r)}{R}\omega_1. \quad (10)$$

One can see that Eqs (3, 10) transform to Eqs (8, 9) for $r<<R$. The designated two fundamental errors from Ref. (9, 10), which nevertheless direct to a result identical to the one presented in this paper, create an illusion that Eqs (8), (9) can be used for a ball rolling on a curved surface. Moreover, from a pedagogical point of view, this in fact has a negative outcome as it is ignoring the need for a critical judgment to evaluate the appropriateness of a direct transfer of equations (such as (8), (9)) applicable in some problems to other situations that are not the same. It should also be mentioned that it is a typical error to use Eqs (8), (9) for a ball rolling on a concave surface as it has also been done for finding the period of a ball's oscillations rolling back and forth on a concave surface [11].

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Intrinsic Nonlinearity of a PN-junction Diode and Higher Order Harmonic Generation

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Abstract

Voltage current characteristics of a PN-junction diode are intrinsically nonlinear in nature. It is shown in this paper that a mathematical form of nonlinearity of a PN-junction diode resembles the nonlinear response of electric polarization of a dielectric medium to the electric field. Nonlinearity of a PN-junction can be expressed in a series of successively increasing orders of the nonlinearity. For a PN-junction diode, higher order nonlinear terms become significant as voltage across the diode is increased. In this paper, a gradual emergence of a nonlinear regime with the amplitude of a sinusoidal voltage is presented. Higher order harmonics are produced by utilizing the nonlinearity of a single PN-junction diode. An experimental realization of a frequency comb with the highest frequency up to the twentieth harmonics is also presented. In addition, in the same circuit by making the nonlinearity significant up to the second order, an experiment on generation of the sum and difference of frequencies is realized.

1 Introduction

Nonlinearity is present in most of the natural processes. A linear behaviour of a system is an approximation of a generalised nonlinear response [1]. Whether it is a simple pendulum or interaction of light with matter, a large amplitude of oscillation leads to a breakdown of a linear approximation and brings out the nonlinear response of a system. Nonlinear physics is a well established field of research. Some of the prominent concepts such as the emergence of chaos [2] and in nonlinear optics where light fields interact with each other [4, 5, 3] have been of great interest. In this paper, an experiment based on a PN-junction diode is introduced which utilizes the intrinsic nonlinearity of a PN-junction diode to experimentally realize the effects analogous to those observed in classical nonlinear optics. The voltage current characteristics of a PN-junction diode can be expressed in the form of an infinite series of linear and nonlinear

terms. With an increase in the amplitude of a sinusoidal voltage applied across a PN-junction diode the higher order nonlinearities become important. As the amplitude of a sinusoidal voltage is increased, new components of frequency appear in current passing through the PN-junction diode. A gradual generation of new frequencies and generation of a frequency comb [12] with the highest frequency up to the twentieth harmonics are clearly demonstrated in the experiment. Further, by using a source of two different frequency sinusoidal voltage waveforms (a sinusoidal voltage has a wave like form in the time domain), the sum and difference frequency generation have been demonstrated experimentally.

2 Nonlinear dielectric medium: Introduction

In a linear dielectric medium in presence of an electric field, the displacement of bound charges from their equilibrium position remains linearly proportional to the applied force (Hooke's law). As a consequence, the induced electric polarization (P) remains linearly proportional to the net electric field (E) such that $P = \epsilon_0 \chi_e E$, where ϵ_0 is the vacuum permittivity and χ_e is the electric susceptibility, which is a measure of the ability of a medium to get polarized. Therefore, when light propagates in a 'linear' dielectric medium, the frequency of the propagating light wave and the refractive index of the medium remain unaltered. As a consequence, different electromagnetic fields do not interact with each other. However, a linear response is an approximation and as the intensity of light (electric field) is increased, the displacement of bound charges increases. Hooke's law does not remain valid for a large displacement of bound charges and hence the induced polarization does not remain linearly proportional to the electric field. The dependence of the induced electric polarization on the electric field becomes nonlinear which can be expressed as [4, 6, 5]

$$P = \epsilon_0(\chi_e^{(1)}E + \chi_e^{(2)}E^2 + \chi_e^{(3)}E^3 + \cdots) \quad (1)$$

Where $\chi_e^{(1)}$, $\chi_e^{(2)}$ and $\chi_e^{(3)}$ are the first, second and third order electric susceptibilities respectively, the order of nonlinearity is denoted by a superscript. Equation. 1 can be rewritten as the sum of a linear and a nonlinear term such that $P = \epsilon_0\chi_e^{(1)}E + P_{NL}$, where P_{NL} is the nonlinear component of the electric polarization. At a low light intensity, only the first term is significant and all the nonlinear terms are negligible. For a high intensity of light, the P_{NL} term becomes pronounced. The relative magnitude of the nonlinear susceptibilities in Equation 1 depends on the crystal symmetry [7]. For non centro-symmetric crystals such as Lithium Niobate ($LiNbO_3$), Potassium Titanyl Phosphate (KTP) and Beta Barium Borate (BBO) the second order nonlinearity $\chi_e^{(2)}$ is nonzero. A nonlinear medium having only the second order nonzero nonlinearity is known as a second order non-

linear medium [4, 6, 5]. One of the important processes resulting from the second order nonlinearity ($\chi_e^{(2)}$-process) is the second harmonic generation [8], where frequency of light is doubled. According to the quantum description of second harmonic generation, two photons of frequency ω interact with each other via the second order interaction and merge together as a single photon of frequency 2ω. In a reverse process which is known as frequency down conversion, a photon can split into two photons of lower frequencies. Other examples of second order nonlinear processes are the sum and difference frequency generation [9, 10, 4, 6, 5, 11, 12]. For a medium such as the silica optical fiber, $\chi_e^{(3)}$ is significant which results in third order nonlinear effects [13, 14], where the refractive index becomes intensity dependent - a phenomenon known as the optical Kerr effect. Optical Kerr effect can be utilised to modulate the phase of a wave by modulating the intensity of another wave. A few important examples of third order nonlinear processes [6, 14] are the third harmonic generation [15], self phase modulation, cross phase modulation, four wave mixing, supercontinuum generation and optical phase conjugation. These nontrivial effects have revolutionized the field of nonlinear optics over the decades and have played a significant role to produce quantum entangled photons and in experiments on quantum information processing.

3 Nonlinearity of a PN-junction diode

An ideal diode conducts current in the forward bias and completely blocks the flow of current in the reverse bias. A PN-junction is formed, if a P-type semiconductor is joined to a N-type semiconductor. Immediately after joining them, electrons which are majority carriers of N-type region start diffusing into P-type region due to their concentration gradient across the contact of two semiconductors. Diffused electrons recombine with holes which are majority carriers of P-type region. Flow of charge carries through the contact due to concentration gradient generates a diffusion current. However, this process is gradually stopped by an opposing force. As electrons diffuse into the P-type region, a net positive charge develops near the contact in N-type region and a net negative charge is formed near the contact in P-type region. This charge distribution produces an electric field in the PN-junction pointing from N-type region towards P-type region. Electric field generates an electric potential barrier at the junction and an opposing force for the flow due to diffusion of majority carriers. However, minority carriers which are produced by thermal agitation, are accelerated by the junction electric field. This process produces current named as the drift current. In equilibrium condition, if a PN-junction is unbiased, the diffusion current and drift current flow in opposite direction. Therefore, a net flow of current is zero

through a PN-junction *i.e.* majority carriers climb the potential barrier and minority carriers descends the potential barrier. Under the forward bias, P-type region is kept at a higher potential as compared to N-type region by connecting to a battery. In the forward bias, the net electric field and height of potential barrier across the junction is decreased. Therefore, probability of carriers to climb the potential barrier increases in the forward bias. This process increases electron concentration in P-type region at the junction edge and hole concentration in N-type region at the junction edge. Such an excess carrier concentration increases exponentially with an applied bias voltage across the junction. Therefore, in the forward bias, the diffusion current increases exponentially with applied bias voltage while the drift current remains negligibly small. In the reverse bias, P-type region is kept at a lower potential as compared to N-type region. Therefore, potential barrier height increases and probability to cross the potential barrier decreases. However, the flow of drift current saturates with the increase in the reverse bias voltage due to an extremely low carrier concentration of thermally generated minority carriers. The flow of net current through a PN-junction is a nonlinear function of the bias voltage. However, in case of a nonlinear dielectric medium, it is the displacement of charge from the equilibrium position which depends nonlinearly on the applied force. Nonlinear dielectric medium and a PN-junction diode are analogous to each other in the context of a mathematical form of their nonlinearity, which is the central idea of this paper.

In a realistic PN-junction diode, for a bias voltage V across a junction the net current I passing through the junction is [16]

$$I = I_0(e^{V/\eta V_{th}} - 1) \tag{2}$$

Where I_0 is the reverse saturation current, η is the ideality factor which is constant (value between 1 to 2) and $V_{th} = \frac{k_B T}{q}$ is the thermal voltage, k_B is the Boltzmann constant, T is the absolute temperature and q is the charge on electron. A typical numerical value of the thermal voltage is 25.85 mV at 300 K. According to Equation. 2, a PN-junction diode current increases exponentially when a forward bias voltage V is increased, while under a reverse bias the current saturates to I_0.

Taylor series expansion of Equation. 2 can be written as

$$I = I_0\left(\frac{V}{\eta V_{th}} + \frac{1}{2!}\frac{V^2}{\eta^2 V_{th}^2} + \frac{1}{3!}\frac{V^3}{\eta^3 V_{th}^3} + \frac{1}{4!}\frac{V^4}{\eta^4 V_{th}^4} + \frac{1}{5!}\frac{V^5}{\eta^5 V_{th}^5} + \dots\right) \tag{3}$$

Which can be expressed as

$$I = \chi_v^{(1)} V + \chi_v^{(2)} V^2 + \chi_v^{(3)} V^3 + \chi_v^{(4)} V^4 + \chi_v^{(5)} V^5 + \dots \tag{4}$$

Where $\chi_v^{(1)} = I_0/\eta V_{th}$, $\chi_v^{(2)} = I_0/2!\eta^2 V_{th}^2$, $\chi_v^{(3)} = I_0/3!\eta^3 V_{th}^3$, and so on. Where a superscript denotes the order of nonlinearity.

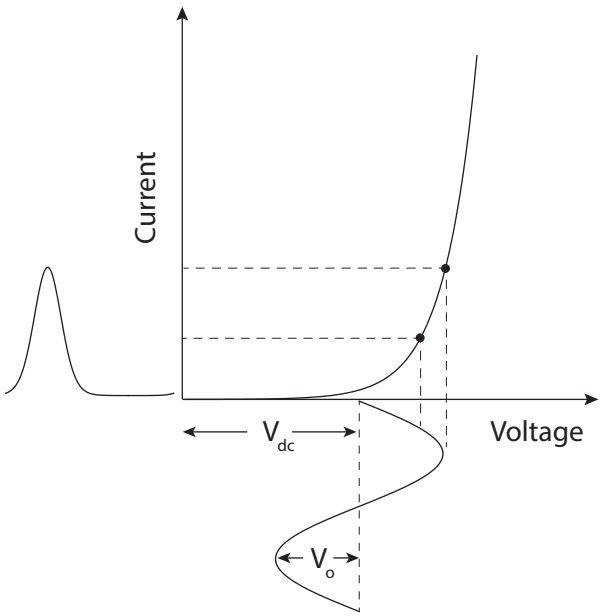

Figure 1: A schematic showing an exponential dependacnce of current on the forward bias voltage across a PN-junction diode. A variation of diode current for a sinusoidal bias voltage of amplitude V_o with a dc offset V_{dc} is also shown.

The first term of Equation. 4 is linear and all the remaining terms represent a nonlinear dependence of current on voltage. Equation. 4 is analogous to Equation. 1 where the voltage is analogous to the electric field and the current is analogous to the electric polarization.

The voltage current characteristics of a PN-junction diode are qualitatively presented in Figure. 1 where an exponential rise of current with the bias voltage is shown. A sinusoidal voltage of amplitude V_o with a constant dc-offset voltage V_{dc} is applied across the diode. Because of the nonlinear characteristics of the PN-junction diode the current is nonsinusoidal. For a positive dc-offset voltage such that $V_{dc} > V_o$, the diode is always forward biased. However, for $V_{dc} = 0$ the diode is reverse biased during the negative half cycle and consequently a half wave rectification occurs. A circuit designed to study the nonlinearity of a PN-junction diode is shown in Figure. 2. This circuit produces a voltage output which is linearly proportional to current passing through the diode. Input voltage of the circuit is applied parallel to the diode. Circuit consists of two 741 general purpose operational amplifiers and a PN-junction diode IN4007 which is a nonlinear element in the circuit. The first operational amplifier is configured in an inverting voltage adder configuration whose output voltage is a summation of the input voltages with reversed polarity ($-(V_s + V_{dc})$). The output of the voltage adder is connected to the N-type terminal of a PN-junction diode (IN4007) while the P-type terminal is connected to the inverting terminal of a second operational amplifier. A typical numerical value of reverse saturation current I_o of the PN-junction diode-IN4007 is 5.0 μA. Since an inverting terminal of the second operational amplifier is biased at the virtual ground potential, therefore, the PN-junction diode becomes forward biased if the output of the voltage adder is negative. The second operational amplifier is operating in an inverting current-to-voltage converter configuration and its output voltage is $V_t = R_F I$, where I is the current flowing through the

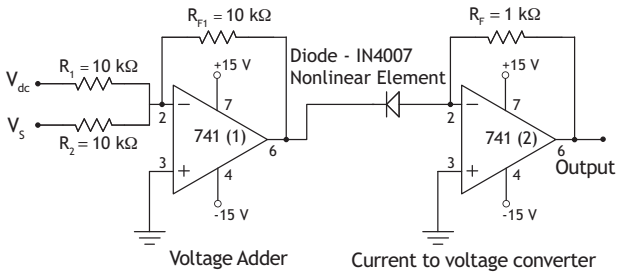

Figure 2: A diagram of a circuit designed to study the nonlinearity of a PN-junction diode. Two 741 general purpose operational amplifiers with a PN-Junction diode IN4007 are used in the circuit. Output of an inverting voltage adder is applied across a PN-junction diode. The output voltage of the circuit is proportional to current passing through the PN-junction diode.

PN-junction diode. Therefore, the output voltage of the current-to-voltage converter is $V_t = R_F I_0(e^{(V_s+V_{dc})/\eta V_{th}} - 1)$. Final output voltage V_t of the circuit and voltage across the PN-Junction diode (potential at the N-type terminal where the P-type terminal is connected to a virtual ground) is shown in Figure. 3, where $V_{dc} = 360$mV and $V_o = 180$mV.

Nonlinear response of the circuit is clearly shown in the upper plot of Figure. 3 where the current passing through the diode (converted to voltage by an operational amplifier) is non sinusoidal for a sinusoidal input voltage. The output voltage waveform is non-sinusoidal therefore, it shows the presence of more than one frequency components.

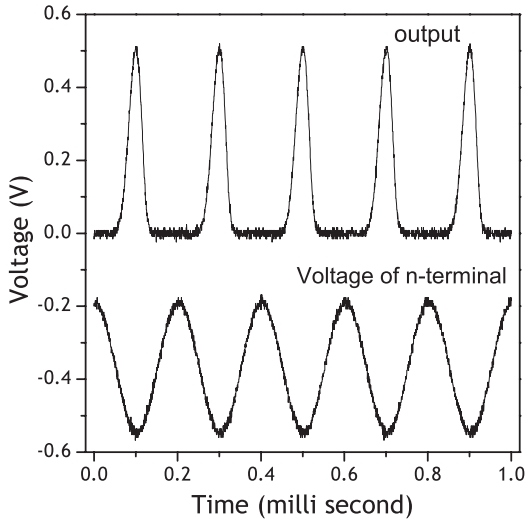

Figure 3: A plot showing an applied sinusoidal voltage across a PN-junction diode (lower plot). Output voltage V_t (upper plot) is linearly proportional to current passing through the diode. It is clearly shown that a sinusoidal input voltage gives rise to a non-sinusoidal output voltage due to the intrinsic nonlinearity of the diode.

4 Phase Matching

In nonlinear wave mixing processes, conditions of energy and momentum conservation should be fulfilled. The first condition implies that the total energy of interacting fields must be same before and after the nonlinear interaction. If all the waves at different spatial points at an instant of time in an extended medium are in phase with each other then a constructive interference occurs. This results in a high efficiency of harmonic generation. Any deviation from an exact phase matching results in a decrement of efficiency of harmonic generation. A phase matching condition implies conser-

vation of momentum in an extended nonlinear medium. In case of a PN-junction diode, the nonlinearity is not extended however, its source of origin is localized at the junction. For a typical frequency of oscillating voltage waveform the wavelength of oscillating voltage waveform in circuit is much larger than the extension of the circuit. A nonlinear PN-junction is equivalent to a single nonlinear dipole and in this case the condition of phase matching implies resonance of driving field with the oscillator modes. In case of an extended nonlinear medium the nonlinearity is a collective process and coefficients of the orders of nonlinearity are governed by symmetry of a nonlinear crystal however, in case of a single dipole the notion of symmetry disappears. It is shown in this paper that all nonlinear terms appear for a PN-junction diode in a nonlinear regime.

5 Emergence of Nonlinear Regime

We now move to the experiment to demonstrate a generation of new frequencies from the nonlinear response of the PN-junction diode. The output voltage V_t of the circuit shown in Figure. 2 can be written as

$$V_t = R_F(\chi_v^{(1)}V + \chi_v^{(2)}V^2 + \chi_v^{(3)}V^3 + \chi_v^{(4)}V^4 + \chi_v^{(5)}V^5 + \ldots) \quad (5)$$

Where a voltage applied at input of circuit is $V = V_{dc} + V_s$ and $V_s = V_o\cos(\omega_o t)$ is a sinusoidal voltage corresponding to a fundamental harmonic of angular frequency ω_o. For a low input voltage $V_o + V_{dc}$ the linear term is the only significant term, therefore, the output voltage V_t is $R_F\chi_v^{(1)}(V_{dc} + V_o\cos(\omega_o t))$, which has the same frequency spectrum as the spectrum of the input waveform, namely the presence of a single frequency ω_0. If the amplitude of the input sinusoidal voltage V_o is increased such that only the second order term of Equation. 5 becomes significant and all the remaining nonlinear terms are negligible then the total output voltage due to the first two terms of Equation. 5 can be written as

$$V_t = R_F\Big(\chi_v^{(1)}V_{dc} + \chi_v^{(2)}(V_{dc}^2 + \frac{V_o^2}{2}) + (\chi_v^{(1)} + 2\chi_v^{(2)}V_{dc})V_o\cos(\omega_o t) + \chi_v^{(2)}\frac{V_o^2}{2}\cos(2\omega_o t)\Big) \quad (6)$$

The voltage waveform V_t which is proportional to the current passing through the PN-junction diode contains a new frequency component ($2\omega_o$) of frequency twice the frequency (ω_o) of the input sinusoidal voltage V_s. The generation of new frequency component corresponds to the second harmonic generation. A further increase of amplitude V_o reinforces the third order nonlinear term to be significant in addition to the second order term. The contribution to the output voltage V_t due to the third term of

Equation. 5 is written as

$$R_F\chi_v^{(3)}\Big(V_{dc}^3+\frac{3}{2}V_{dc}V_o^2+3\big(V_{dc}^2V_o+\frac{V_o^3}{4}\big)\cos(\omega_o t)+\frac{3}{2}V_{dc}V_o^2\cos(2\omega_o t)+\frac{V_o^3}{4}\cos(3\omega_o t)\Big) \quad (7)$$

which has frequency components at zero, ω_o, $2\omega_o$ and $3\omega_o$. The harmonics of frequency $3\omega_o$ corresponds to a third harmonic generation. The total output voltage V_t is a summation of contribution from the first three terms of Equation. 5. Similarly, the contribution from the fourth order nonlinear term of Equation. 5 to the output voltage V_t is written as

$$R_F\chi_v^{(4)}\Big(V_{dc}^4+3V_{dc}^2V_o^2+\frac{3V_o^4}{8}+(4V_{dc}^3V_o+3V_{dc}V_o^3)\cos(\omega_o t)+\big(3V_{dc}^2V_o^2+\frac{V_o^4}{2}\big)\cos(2\omega_o t)+V_{dc}V_o^3\cos(3\omega_o t)+\frac{V_o^4}{8}\cos(4\omega_o t)\Big) \quad (8)$$

which has frequency components at zero, ω_o, $2\omega_o$, $3\omega_o$ and $4\omega_o$. The frequency component at $4\omega_o$ signifies the fourth harmonic generation. The total output voltage V_t is again an addition of contributions from all the significant terms in Equation. 5. A gradual emergence of a nonlinear regime is observed by generation of higher order harmonics as the amplitude of the input sinusoidal voltage V_s is increased. A spectrum analyzer is connected to the output of the circuit to record the frequency spectrum of the output voltage waveform. The output voltage is also plotted on a digital oscilloscope to record the signal in time domain. A series of plots shown in Figure. 4 shows a gradual appearance of higher order harmonics as the amplitude of the sinusoidal input voltage is increased. The fundamental frequency of the input sinusoidal voltage waveform is chosen to be $f_o = \omega_o/2\pi = 5\,kHz$ and the offset voltage is V_{dc} = 360 mV in all the plots. Figure. 4 (a) corresponds to a linear regime, where the amplitude of higher harmonics is negligibly small. A plot in Figure. 4(b) shows the emergence of the nonlinear regime, where the second order nonlinearity becomes significant, and a second harmonics at frequency 10 kHz is observed. Another plot in Figure. 4(c) shows a regime where the nonlinearity up to the third order is pronounced as shown by a third harmonic generation at frequency 15 kHz. A plot in Figure. 4(d) shows a regime where the nonlinearity up to the fourth order is pronounced as shown by a fourth harmonic generation at frequency 20 kHz. Therefore, higher orders of nonlinearity can be addressed by increasing voltage across a PN-junction diode.

5.1 Frequency comb generation

A frequency comb is a series of equally spaced discrete harmonics. In this experiment a frequency comb has been generated by utilizing the nonlinear response of a PN-junction diode. A sinusoidal input voltage with a nonzero voltage offset is applied to the input of the circuit as shown in Fig-

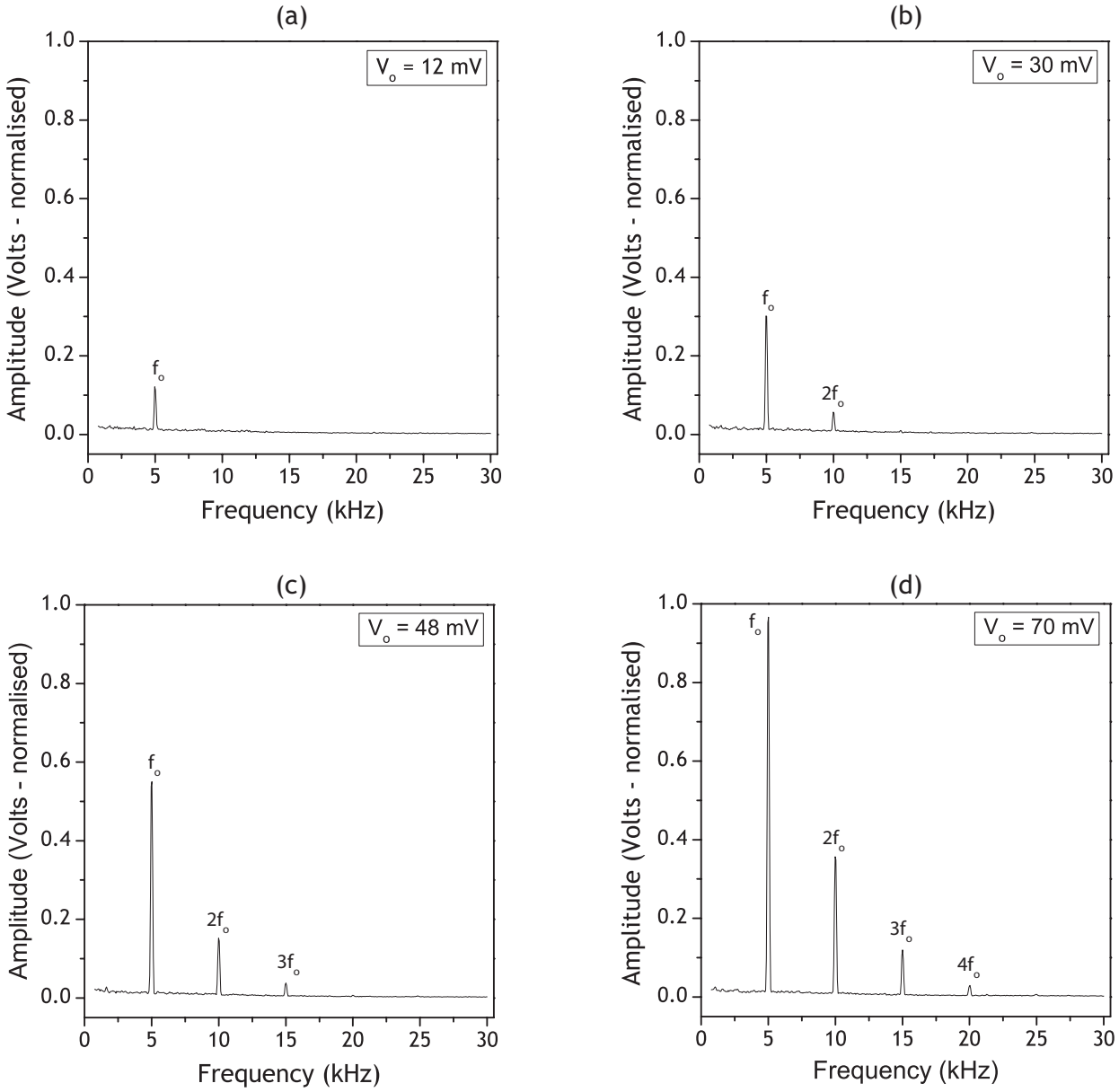

Figure 4: A series of plots showing a gradual emergence of a nonlinear regime of higher orders as the amplitude of the input voltage waveform is increased. The amplitude of the input sinusoidal voltage is shown on each spectrum. The dc-offset is constant and frequency of the input sinusoidal voltage is 5 kHz in all plots. Plot (a) shows a linear regime where no new frequency is generated. (b) Represents an emergence of the nonlinear regime where the second harmonic generation is observed. (c) A third order nonlinearity is also pronounced and frequency components up to the third harmonics are observed. (d) A fourth order nonlinearity is gradually pronounced and a frequency component of fourth harmonics is observed.

ure. 2. The output voltage is observed in time domain on a digital oscilloscope. The amplitude of the input sinusoidal voltage of frequency 5 kHz is gradually increased until the output similar to the output voltage shown in Figure. 3 is observed. A frequency spectrum analyzer is connected to the output of the circuit to measure a frequency spectrum of the output voltage. For a large amplitude the higher order nonlinear terms are pronounced and a comb of frequencies with the highest frequency up to the twentieth harmonics of frequency 100 kHz is observed, where the frequency of input sinusoidal voltage is 5 kHz. An electrical power spectrum of the output voltage is shown in Figure. 5, where the measured electrical power is indicated on a logarithmic scale (1 dBm= $10\log_{10} P/1mW$ and P is measured in milli-Watt (mW)). This is essential to keep the amplitudes corresponding to different frequency components within the scale margins. Frequency combs are useful in many areas of science and this demonstration introduces the idea in a simple system.

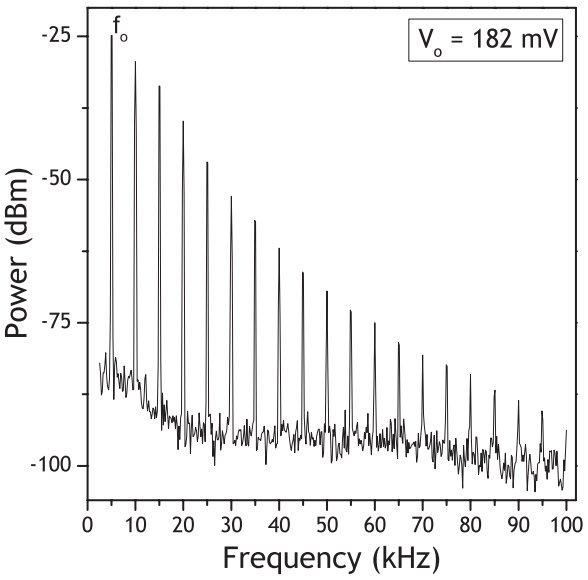

Figure 5: A frequency spectrum of a frequency comb. A highest frequency 100 kHz corresponds to the twentieth harmonics and frequency of input voltage waveform is $f_o = 5\,kHz$.

5.2 Sum and difference frequency generation

The sum and difference of frequencies of different sinusoidal voltage waveforms can be produced from the second order nonlinearity. Two independent sinusoidal voltage waveforms V_{s1} and V_{s2} with a constant dc voltage offset are applied at the inputs of the circuit shown in Figure. 2. A second sinusoidal voltage waveform V_{s2} is connected to an inverting terminal of the first operational amplifier through an additional 10 kΩ resistance (not shown in the circuit diagram). In this case the total input voltage is $V = V_{dc} + V_{s1} + V_{s2}$ where, $V_{s1} = V_{o1}\cos(\omega_1 t)$ and $V_{s2} = V_{o2}\cos(\omega_2 t)$. The output voltage is given by Equation. 5. Consider the waveform amplitudes V_{o1} and V_{o2} are such that only first two terms of the Equation. 5 are significant *i.e.* only the second order nonlinearity is significant. Therefore, the total

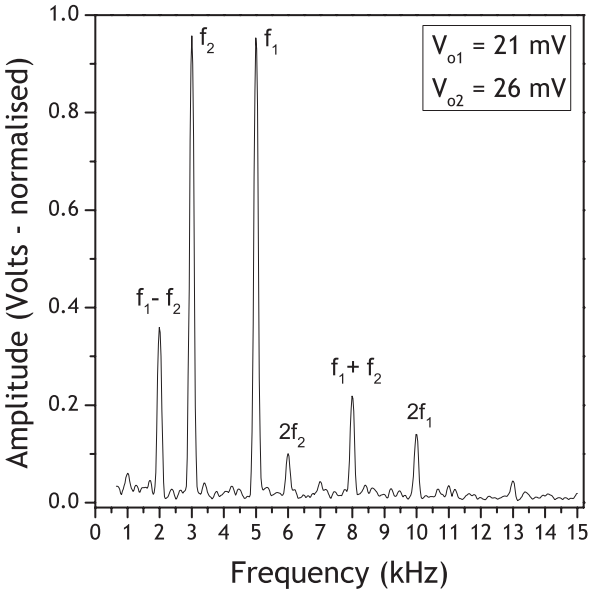

Figure 6: A frequency spectrum of the output voltage waveform shows the sum ($f_1 + f_2$) and difference ($f_1 - f_2$) frequency generation. Second harmonics of frequencies $2f_1$ and $2f_2$ corresponding to each input voltage waveforms are also produced.

output voltage of the circuit is written as

$$V_t = R_F I_0 \Bigg(\frac{\chi_v^{(2)}}{2}(V_{o1}^2 + V_{o2}^2 + 2V_{dc}^2) + \chi_v^{(1)} V_{dc} + (\chi_v^{(1)} V_{o1} + 2\chi_v^{(2)} V_{o1} V_{dc}) \cos(\omega_1 t) + \frac{\chi_v^{(2)}}{2} V_{o1}^2 \cos(2\omega_1 t) + (\chi_v^{(1)} V_{o2} + 2\chi_v^{(2)} V_{o2} V_{dc}) \cos(\omega_2 t) + \frac{\chi_v^{(2)}}{2} V_{o2}^2 \cos(2\omega_2 t) + \chi_v^{(2)} V_{o1} V_{o2} \cos(\omega_1 - \omega_2) t + \chi_v^{(2)} V_{o1} V_{o2} \cos(\omega_1 + \omega_2) t \Bigg) \quad (9)$$

The output voltage waveform consists of harmonics of frequencies equal to the sum ($\omega_1 + \omega_2$) and the difference ($\omega_1 - \omega_2$) of frequencies of the input voltage waveforms. In addition, there are frequency components at $2\omega_1$ and $2\omega_2$ which correspond to second harmonic generation corresponding to the individual voltage waveforms V_{s1} and V_{s2}, respectively. A measured frequency spectrum of the output voltage waveform V_t is shown in Figure. 6 for $f_1 = \omega_1/2\pi = 5\ kHz$ and $f_2 = \omega_2/2\pi = 3\ kHz$ where, $V_{o1} = 21\ mV$ and $V_{o2} = 26\ mV$ are chosen such that only the second order nonlinearity is significant. A measured spectrum explicitly shows the sum of frequencies ($f_1 + f_2 = 8\ kHz$) and the difference of frequencies ($f_1 - f_2 = 2\ kHz$). In addition. the measured spectrum contains frequency components of second harmonic generation at $2f_1 = 10\ kHz$ and $2f_2 = 6\ kHz$ corresponding to the individual voltage waveforms V_{o1} and V_{o2}, respectively.

6 Conclusions

In summary, we have explored the nonlinearity of voltage current characteristics of a PN-junction diode and experimentally demonstrated various concepts related to nonlinear physics in particular nonlinear optics. Taylor expansion of the voltage current characteristics of a PN-junction diode resembles the nonlinear dependence of electric polarization of a dielectric medium on the electric field which allows an analogy between the two systems.

A gradual emergence of the nonlin-

ear regime of successively increasing orders has been shown experimentally. A frequency comb with the highest frequency up to the twentieth harmonics has also been produced. In addition, an experiment to create the sum and difference of frequencies of two independent sinusoidal voltage waveforms is presented. It may also be possible to observe certain other aspects of nonlinear physics using the *PN*-junction diode experiment and these aspects will be presented elsewhere.

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Kinematics of a Falling Slinky

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Abstract

Experimental observation of the speed of the top coil of a falling Slinky exhibits two non-intuitive features: it reaches a maximum speed shortly after release, and then the speed decreases until full collapse. The paper's purpose is to determine the physical processes and reasons for the non-intuitive features.

1 Introduction

An earlier publication displays a plot of the top coil's speed of a falling Slinky as a function of time [1]. The two non-intuitive speed features are: first, there is a maximum speed of $5.6 \pm .4$. m/s in the 0.02 - 0.04 sec. interval after release and second, the speed decreases until full Slinky collapse. At which point, the speed is equal to that of a point mass falling thru the displacement from the Slinky's center of mass to the bottom coil. The experimental paper's authors used a finite-collapse time model associated with longitudinal wave front propagation to obtain a qualitative curve fit to the speed profile by adjusting the model's single spring coil constant. However, they do not address the physical causes for the maximum speed nor its reduction during collapse.

The paper's purpose is to address the physical processes and reasons for the two aforementioned non-intuitive behaviors. The reasons are discussed in terms of first principles and the quantities associated with the collapsing coil's motion, such as position, speed, acceleration, momentum, single coil mass and spring constant. the Results section discusses and displays the experimental and calculated speed profiles. The Methods section introduces the Slinky center-of-mass model (hereafter COM) used to determine the Slinky's parameters, and discusses the mathematical modeling of the collision kinematics. The Conclusion and Discussion section deals with a single fractional speed difference which lets us under-

stand the increase, the maximum, and the decrease of the top coil's speed.

2.0 Results

The experimental and calculated speed profiles. Two Slinkys were used in the experimental speed measurements, a standard metal Sliny and a plastic Slinky. We confine discussion to the 200 gram metal Slinky which has a hanging length of 1.0 m, 72 free coils and an additional 8 coils closely grouped the bottom. The hanging Slinky was dropped from rest and videos taken at 300 frames/sec. The coil position is determined each 0.01 sec and the speeds experimental uncertainty [2] is $\pm$ 0.4 m/s.

Two speed profiles versus time are displayed in Figure 1. The open circles represent the calculated speed and the dark vertical lines represent the experimental speed. The experimental speed uncertainty of $\pm$ 0.4 m/s is not displayed, the time intervals of 0.025 sec are used for reason of clarity. Between 0.025 and 0.050 sec the maximum speed is 5.6 $\pm$ .4 m/s, and it diminishes at complete collapse to 3 $\pm$ .4 m/s at a time of 0.27 sec.

The profile of circles is the result of computations using the Slinky kinematical analysis discussed in the Methods section. The calculated maximum speed is 5.5 m/s at 0.03 sec between the 7th and 8th coil, and the speed diminishes until complete collapse at 0.31 sec. The model's computations yield a speed of 2.9 m/s at full collapse. From the point of view of the Slinky's COM motion, it has been displaced from rest downwards 0.37 m as the Slinky collapses. The corresponding speed from COM energy conservation is 2.7 m/s, both speeds are within the range of the experimental value of 3 $\pm$ .4 m/s

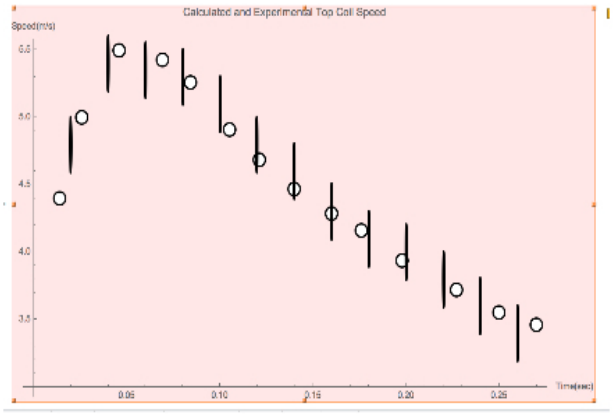

Figure 1 displays both the experimental (vertical bars) and calculated (circles) values for the top coil speed as a function of time. The experimental data point is in the center of the vertical line, their maximum (top) and minimum (bottom) values display only 1/2 their uncertainty which was given as $\pm$ 0.4 m/s

3.0 Methods

The Slinky center-of-mass model. This model is introduced to determine the single coil spring constant and the COM position. The model describes the Slinky as a series of Hooke's Law coils each with a mass

m_c and spring constant k_c, originally proposed by Sawicki[3]. The model is a column of alternating point masses and massless springs. For a given single coil, the COM position is located at the coil's half-way point between the coil's top and bottom.The corresponding massless spring with spring constant k_c is below the point mass.

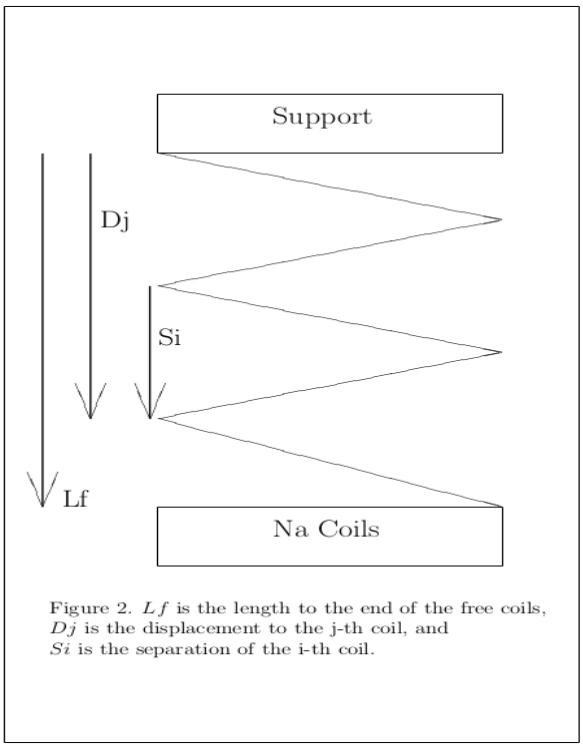

Figure 2. Lf is the length to the end of the free coils, Dj is the displacement to the j-th coil, and Si is the separation of the i-th coil.

3.1 The Slinky Configuration

A schematic diagram of the Slinky is show in Figure 2. The Slinky coils are partitioned into two groups [3]. They are either free with a clear separation between them, or very closely grouped near the bottom where their weight is not sufficient to produce a visible separation between adjacent coils. The 72 free coils are denoted by N_f and the remaining 8 grouped coils are denoted by N_a. The details of the COM model are fully discussed elsewhere so the discussion here is brief.[4]

The top -to- bottom separation of the i-th coil is denoted by S_i. Given that each coil supports all those below it, an application of Hooke's Law yields the equation:

$$k_c\ S_i = m_c\ \mathrm{g}\ (\ N_f + N_a - \mathrm{i}\ ) \qquad (1)$$

We solve for the coil separation an obtain

$$S_i = (\ \mathrm{g}\ m_c/k_c)\ (\ N_c + N_a - \mathrm{i}\ ) \qquad (2)$$

The displacement from the support to the bottom of the j-th coil is denoted by D_j, and it is the sum of all the S_i values from i=1 to i=j. Using the summation identity for integers we obtain

$$D_j = (\mathrm{g}\ m_c/k_c)((N_f+N_a)\ \mathrm{j} - (\mathrm{j}/2)(\mathrm{j}+1)) \quad (3)$$

The free hanging length of the Slinky. L_f from the support to the bottom of the. N_f free coils is D_j where j=N_f. For L_f = 1.00 meter, N_f= 72 and N_a=8, we obtain k_c= 77 N/m using Eq.(3) with the single coil mass m_c=0.0025 kg. The corresponding single coil frequency is 27.8 Hz with a period of 0.036 sec. In the experimental paper the spring constant was used as an adjustable parameter with a value of 55 N/m to obtain the best fit to the data.

3.2 Observational basis for the kinematic model.

The kinematic model is based upon an interesting observed behavior: not all coils uniformly accelerate downward after release. *It is observed the j-th coil remains at rest until the column of j-1 collapsed coils above falls upon it.*

3.3 Kinematic Collision Model for j-collaped coils.

Hence, a simplified model is a repetitive series of collisions of the j-collapsed coils colliding the j+1-th coil at rest. The collision forms a new set of j+1 collapsed coils and the process continues until full collapse. The results below show the simplified model is sufficient to explain both two non-intuitive collapse features.

The j-collapsed coils' COM is accelerated thru the separation of the j+1th coil, S_{j+1} which increases the j-collapsed coils' speed. Conservation of momentum in the collision reduces the speed of the j+1 collapsed coils. The process is repeated until all the coils have collapsed. In each collision we are interested in the speed after the collision, as we take it to be equal to the speed of the top coil since all collapsed coils are moving together.

3.4 Collision of the j-collapsed coils with the resting j+1-th coil

The simplified model is a two step process. First, the collapsed j-coils COM moving with an initial velocity $V_{0,j}$ undergoes an acceleration a_{j+1} thru a distance S_{j+1} to the j+1-th coil where its increased velocity is denoted by $V_{1,j}$. The relationship between the two velocities is given by

$$V_{1,j}^2 = V_{0,j}^2 + 2\, a_{j+1}\, S_{j+1} \tag{4}$$

Second, in the collision the j+1-th coil is initially at rest, as previously noted. Momentum is conserved and the velocity of the j-collapsed coils is reduced and represented by $V_{0,j+1}$. The downwards direction is taken as positive. The relationship between the j-collapsed coils' final speed $V_{1,j}$ and the initial speed of the j+1 collapsed coils is given by

$$V_{0,j+1} = (\,j/(j+1))\; V_{1,j} \tag{5}$$

3.5 Acceleration of the j-collapsed coils

The acceleration is determined by summing all the forces acting upon the j-collapsed coils. And they are (a) the weight of the j-collapsed coils j m_c g, the force exerted by the weight of the coils below, (N_f+N_a -j)m_c g, and the Hooke's Law force from the j+t–th coil. The sum of the forces is expressed below:

$$j\, m_c\, a_j = j\, m_c\, g + g\, m_c\, (N_f + N_a - j) + k_c\, S_{j+1} \quad (6)$$

We solve for a_j and obtain

$$a_j = (1/j)\,(g\,(N_f + N_a) + (k_c/m_c)\, S_j) \quad (7)$$

We observed that for the last coil when $j = N_f + N_a$, and $S_j = 0$, as expected the acceleration is g.

For example, for the first coil, the separation S_1 given by Eq.(2) is 2.5 cm, the corresponding acceleration from Eq.(7) is 1554 m/s^2, and its speed, $V_{1,1}$ from Eq.(4) before colliding with the second coil is 8.81 m/s. After collision with the second coil, the initial speed of the 2 collapsed coils is 4.41 m/s from Eq.(5). Repetition of this process generated the open circle points on Figure 1.

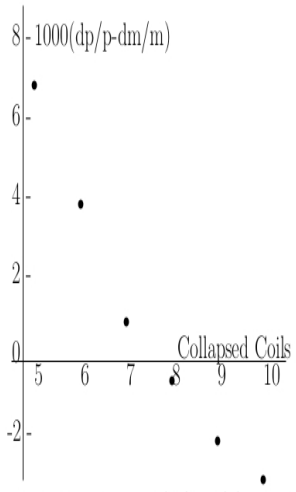

Figure 3. Sign change of (dp/p -dm/m) occurs between coils 7 and 8 at maximum speed

4.0 Discussion

The logical starting point for the physics involved is the momentum of the j-collapsed coils, P_j since its value depends upon both the mass (M_j) of the collapsed coils and their speed V_j which are changing during the fall. Their momentum is given by

$$P_j = M_j\, V_j. \quad (8)$$

Our interest is in obtaining an expression relating the fractional changes in the momentum, the mass and the velocity, and it is given by

$$dP_j/P_j = dV_j/V_j + dM_j/M_j \quad (9)$$

We replace all the differentials by the small increments occurring during one collision interval for the j-collapsed coils, and rewrite the equation to obtain

$$\Delta V_j/V_j = \Delta P_j/P_j - \Delta M_j/M_j. \quad (10)$$

Note that the incremental fractional mass expression

$$\Delta M_j/M_j = 1/j, \quad (11)$$

since $M_j = j\, m_c$ and $\Delta M_j = m_c$. Figure 3 is a plot of the right side of Eq.(10) vs the number of collapsed coils using the kinematic model. The vertical coordinate is enlarged by a factor of a 1000 for ease of plotting.

5.0 Conclusion

First, the top coil's speed reaches its maximum when the incremental speed is zero

when the incremental fractional momentum equals the incremental fractional mass. From inspection of Figure 3, the two are equal somewhere between coil 7 and 8 when the speed, from Figure 1 is 5.5 m/s between 0.03 sec and 0.05 sec. Second, in Figure 3 between collapsed coils 1 and 7 when t ¡ 0.03 sec the incremental fractional momentum (large accelerations and large coil separations) is greater than the incremental fractional mass (1/3, 1/4, ect.); hence from Eq.(10) in incremental fractional speed is positive so the speed increases. Third, in Figure 3, for collapsed coils equal to or greater than 8 when t ¿ 0.05 sec, the incremental fractional momentum (smaller accelerations and coil separations) is less than the incremental fractional mass (1/20, 1/21, 1/22, ect.); hence, from Eq.(10) the fractional speed is negative and the speed decreases.

5.1 Bottom Coil Suspension Times

. The fact the Slinky's bottom coil remains at rest while the top collapses impresses students provided there is sufficient time for observation. The observation is not too common because the suspension time for one standard Slinky (80 coils) is too short (3/10 sec) to observe. When the kinematic model program is run for two Slinkys (160 coils) or three Slinkys (180 coils) in series, the suspension times are 0.6 sec and 0.9 sec, respectively. Both times are long enough to clearly observe the bottom coil remaining at rest until all the collapsed coils fall upon it.

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A Toy Model of Two Dimensional Polarizable Material

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Abstract

In this article, we consider a simple toy model of two dimensional polarizable material consisting of a large number N of non-interacting electric dipoles each of moment μ. However, the moments can align themselves within the plane in only four ($\mathbf{4}$) different ways instead of usual two ways with respect to an applied electric field $\mathbf{E}$. We compute the polarization and orientational contribution of the specific heat of such a system with respect to electric field E and temperature T.

1 Introduction

Polarizability, a property of matter is nothing but the ability to form instantaneous dipoles. It effectively determines the dynamical response of a bound system subjected to an external electric field. Besides, it provides reasonable insight into the internal structure of a molecule. A polarizable material is such a material which can be polarized [1, 2] by an external electric field $\mathbf{E}$. In otherwords, there must be some net dipole moment per unit volume of such a material at a given electric field E and temperature T. Generally, polarizability (dimension is $M^0L^3T^0$) increases as the volume occupied by electrons increases. This is due to the fact that larger atoms have more loosely held electrons in contrast to smaller atoms with tightly bound electrons. Even the larger molecules are generally more polarizable than smaller ones.

Water is a strong polar molecule having dielectric constant of 80, but alkanes and other related hydrophobic molecules are more polarizable. Since alkene and arene possess more electronegative sp^2 carbons compared to the alkane's less electronegative sp^3 carbons, alkanes are the most polarizable molecules [3].

In all these calculations, we use the standard technique of equilibrium statistical mechanics [4–6] more specifically the canonical partition function and from there compute the polarization and specific heat as a function of external electric field E and temperature T. This method can be suitably applied to magnetic system [7, 8] and the dielectric constant [9] of systems.

In this work, we have modelled the two-dimensional polarizable material as a collection of a large number N of non-interacting electric dipoles having each of moment μ, which can be aligned within the plane in *only four* different ways with respect to an applied external electric field **E**. Here out of four, two of them are parallel to the field (directed along and opposite to the field, respectively) and two of them perpendicular. We here completely neglect the relevant kinetic energy of the dipoles and restrict ourselves to the entire energy from the interaction of the dipoles with the given external field E. The whole system as a rule of canonical ensemble is at equilibrium with large heat reservoir having temperature T. In the text book [4] calculation, model adopted is three dimensional one where continuous angle between the alignent with respect to the external field are integrated out.

2.Computation of Polarization and Specific Heat

The interaction energy of an electric dipole with dipole moment $\vec{\mu}$ in an electric field $\vec{E}$ is given by

$$\mathcal{E}_{int}(\theta) = -\vec{\mu} \cdot \vec{E} = -\mu E \cos\theta \qquad (1)$$

The partition function for one of these dipoles belonging in two-dimensional space can be written as

$$Z_1 = \int_0^{2\pi} g(\theta) \exp(-\beta \mathcal{E}_{int}(\theta)) d\theta \qquad (2)$$

where $g(\theta)$ is regarded as the weight of each state allowable in the system (so called degeneracy or density of states) between θ and $\theta + d\theta$ and $\beta = \frac{1}{k_B T}$. We know from the given configurations that the only allowed orientations for each of the dipole are $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$. As a result, we can write down the value of $g(\theta)$ as

$$g(\theta) = \delta(\theta) + \delta\left(\theta - \frac{\pi}{2}\right) + \delta(\theta - \pi) + \delta\left(\theta - \frac{3\pi}{2}\right) \qquad (3)$$

Hence, the partition function in equation (2) simply reduces to

$$\begin{aligned} Z_1 &= \exp(\beta\mu E) + 1 + \exp(-\beta\mu E) + 1 \\ &= 4\cosh^2\left(\frac{\beta\mu E}{2}\right) \end{aligned} \qquad (4)$$

Now, for N number of these dipoles (ignoring the indistinguishability), we obtain the partition function as

$$Z_N = Z_1^N = 4^N \cosh^{2N}\left(\frac{\beta\mu E}{2}\right) \qquad (5)$$

Now, the electric polarization can be computed from the above partition fucntion (5) as

$$\begin{aligned} P(\beta, E) &= \frac{\partial \log Z_N}{\partial(\beta E)} \\ &= N\mu \tanh\left(\frac{\beta\mu E}{2}\right) \end{aligned} \quad (6)$$

The above equation (6) should be contrasted with the text book result (integrated over the continuous angle θ between 0 and π) [4] where one get the expression of polarization as

$$P = N\mu\left[\coth\left(\frac{\mu E}{k_B T}\right) - \frac{k_B T}{\mu E}\right] \quad (7)$$

At very low temperatures ($\frac{\mu E}{k_B T} \gg 1$), the electric polarization becomes independent of both temperature and electric field yielding

$$P(T \to 0) = N\mu \quad (8)$$

While in the opposite limit i.e. high temperature one ($\frac{\mu E}{k_B T} \ll 1$), the expression of the polarization becomes

$$P(T \to \infty) = \frac{N\mu^2 E}{k_B T} \quad (9)$$

This result (9) should be compared with the familiar result ($P = \frac{N\mu^2 E}{3k_B T}$) obtained from equation (7). It is interesting to note that the expressions (8) and (9) are exactly similar like the magnetization of two state spin system in a uniform magnetic field and at a given equilibrium temperature.

The orientational contribution to the specific heat of the system can be computed from the average energy as

$$C_v = \frac{d < E >}{dT} = \frac{d}{dT}\left(-\frac{\partial}{\partial\beta}\log Z_N\right) \quad (10)$$

A simple calculation yields the expression for the specific heat as a function of temperature T and the strength of the external field E as

$$C_v = \frac{Nk_B}{2}\left(\frac{\mu E}{k_B T}\right)^2 \left[\cosh^2\left(\frac{\beta\mu E}{2}\right)\right]^{-1} \quad (11)$$

Since the specific heat is related to the energy fluctuation in energy in canonical ensemble, thus the specific heat is always *positive* in the whole range of temperature and electric field. In the low temperature limit ($\frac{\mu E}{k_B T} \gg 1$), it is easy to note from the expression (11) that the specific heat $C_v \to 0$ satisfying third law of thermodynamics. On the otherhand in the high temperature limit also, a careful analysis reveals that the specific heat $C_v \to 0$. This clearly indicates that there must be an intermediate temperature (from the continuity of the curve) where the specifc heat will show a maximum value. This characteristic feature is common to any two level spin system in statistical mechanics.

As a generalization of the above system, instead of four orientations if the moments align themselves in the plane in six possible directions, then the equation (3) in such a case takes the form

$$\begin{aligned} g(\theta) &= \delta(\theta) + \delta\left(\theta - \frac{\pi}{3}\right) \\ &\quad + \delta\left(\theta - \frac{2\pi}{3}\right) + \delta\left(\theta - \pi\right) \\ &\quad + \delta\left(\theta - \frac{4\pi}{3}\right) + \delta\left(\theta - \frac{5\pi}{3}\right) \end{aligned} \quad (12)$$

Thus, the partition function for such a system can be written in the similar fashion as

$$Z_N(\beta, E) = [2\cosh(\beta\mu E) + 4\cosh(\beta\mu E)]^N \tag{13}$$

It is interesting to note that the sum of the prefactors in front of **cosh** terms in the partition function exactly accounts for the total orientations of the dipoles. The relevant polarization and the specific heat can be calculated from the above expression (13) as a function of temperature and the field. Finally, instead of four and six, if the dipoles are allowed to orient in the plane in eight ways, then the equation (3) will reduce to

$$\begin{aligned} g(\theta) \;=\; & \delta(\theta) + \delta\left(\theta - \frac{\pi}{4}\right) \\ & + \delta\left(\theta - \frac{\pi}{2}\right) + \delta\left(\theta - \frac{3\pi}{4}\right) \\ & + \delta\left(\theta - \pi\right) + \delta\left(\theta - \frac{5\pi}{4}\right) \\ & + \delta\left(\theta - \frac{3\pi}{2}\right) + \delta\left(\theta - \frac{7\pi}{4}\right) \end{aligned} \tag{14}$$

Hence, the partition function can be easily computed as

$$\begin{aligned} Z_N(\beta, E) = \quad & [2 + 2\cosh(\beta\mu E) \\ & + 4\cosh\left(\frac{\beta\mu E}{\sqrt{2}}\right)\Big]^N \end{aligned} \tag{15}$$

All these systems are unable to show the property of spontaneous polarization as $P \to 0$ with $E \to 0$. Thus, they are not ferroelectric but paraelectric only. They however possess non-zero *positve* electric susceptibiity throughout the range of temperature and field. The absence of spontaneous polarization is rooted [7] to the fact that the partition function as well as the free energy is an even function of the external field ($Z_N(\beta, -E) = Z_N(\beta, E)$). Therefore, for a finite system, the polarization must satisfy $P(\beta, -E) = -P(\beta, E)$ and thus confirming $P(\beta, 0) = 0$. Besides, the dipoles are *non-interacting*, therefore it is expected that there should be not any co-operative phenomenon [8] like spontaneous polarization characteristic for ferroelectric material. In recent years, the study of electro caloric effect (ECE) on ferroelectric nanostructures has been focussed [10]. The ECE occurs when an electric field, applied under suitable adiabatic and reversible conditions, can effectively change the temperature of a polarisable material. The ECE seems to have a tremendous potential for solid state cooling technology for a broad range of applications such as on-chip cooling and temperature regulation for sensors and electronic device.

3. Conclusion

In this paper, we have explicitly computed the polarization and the specific heat of a two dimensional polarizable material in an external electric field and at a given temperature where the dipoles orient themselves in four directions with respect to the applied field. The technique shown to compute the partition function can be generalized to

systems having dipole orientations six and eight in the plane.

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Students' Attitudes towards Physics Practical at the Undergraduate (College) Level: A Study of the Colleges in Meghalaya.

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Abstract

The Laboratory is the one place in the college or an institution where the student spent a large fraction of his/her time, trying to establish what is there to be done as prescribed by the curriculum and also to perform the experiments on a wide range of phenomena. Physics practical for that matter includes a large range of experiments from all the different subjects of Physics. The time spent in the laboratory is the time during which the students are (supposedly) expected to learn through hands-on interaction with the different instruments, through mutual student interactions and also to not only learn but also thoroughly understand the various concepts related to a particular experiment, to be able to apply the same kind of scientific approach later when they are doing their research work(s). It is therefore of utmost importance that the students should have and develop a positive attitude towards the laboratory works so as to enable them to not only learn and understand but also to be able to incorporate new ideas (which is the very essence of any research) into a particular experiment that would in turn greatly benefit them when they enter the realms of research. The aim of this study is to investigate if, whether, there are any such negative attitude/attitudes amidst the students in the state of Meghalaya regarding the Physics laboratory work(s).

1. Introduction

Science is different from other subjects. It is not just the subject of science that is different; in fact the entire process of doing science is different. The means by which knowledge is acquired is different in science than it is in history or mathematics or poetry or etc. Science is different because the answers/solutions to scientific questions/problems are not found in a textbook or through pondering high and lofty thoughts. Indeed, scientists do ponder and think high and lofty thoughts; and indeed students in science classes will find answers in a textbook. But the basis of what scientists believe and why they believe is not the result of mere thinking or reading in a textbook. The basis of what scientists believe is the result of the careful collection and analysis of data obtained from laboratory works and evidences. In any physics class, the beauty of the differentness of science will be most evident when the time for the laboratory comes.

In any physics curriculum, the laboratory is a central and integral part which cannot be done away with. More than just a place in the back of the classroom, the laboratory is the place where physics students learn by doing physics. It is in the laboratory that physics students learn to practice the systematic activities adopted by scientists — asking questions, performing procedures, collecting data, analyzing data, answering questions, and thinking of new questions to explore. According to the results of thorough research done by science educators in many developed countries, practical work plays a very important role in teaching and learning science effectively and efficiently. Over the last thirty-five years, laboratory work has been gaining a central and distinctive place in science education, and science educators have suggested that there are huge benefits in learning science with the help of laboratory activities (Hofstein and Lunetta, 2003; Tiberghen, Veilard, Marechal, Buty and Millar, 2001). In particular, in developed countries the laboratory is used as an important medium of instruction in introductory physics courses.

In the current era, the laboratory is especially important as student-centered inquiry has re-emerged as a modern style in teaching and learning science. However, according to Tobin (1990) learning in the laboratory is meaningful only if the students are given adequate opportunities to manipulate real equipment and materials in an environment suitable for them to construct their knowledge of phenomena and related scientific concepts. However, laboratory work demands a significant amount of time and many times rather expensive instruments. Although laboratory experiments and activities are acknowledged as being fundamental to the teaching-learning process in physics, however, it is a big challenge to organize them efficiently so as to be meaningful for the students to perform the lab work.

An experiment is mostly a controlled quantitative investigation — controlled due to the fact that (usually) the various quantities entering into the experiment are under the control of the experimenter and quantitative in the sense that numerical data are obtained from the various observations taken. There is nothing mysterious as far as an experiment is concern: the investigator ordinarily proceeds according to the scientific method listed for the particular experiment. There are several ways in which the students may expect to benefit from the laboratory work. It helps them to understand and remember the physics they have studied and learned; it gives them practice in the application of physical laws and logic to real cases and in that way helps them to think clearly and logically; and it gives them important skills in the use of scientific instruments and techniques.

One of the major drawbacks of the lab work, however, is due to the paucity of time. A whole year's course adds up to less than six weeks of actual laboratory time (the Ph.D. candidate, on the other hand, ordinarily spends about two years of full-time laboratory work on a single problem) so that they cannot expect to get any thorough mastery of certain specialized laboratory techniques; however, they usually learn about less specialized techniques. Most of the principles of physics though, were discovered by men using instruments no better than the simple laboratory instruments. Most of the instruments used, in fact, were not as good. It is a general observation that the students and teachers have to spend large amount of time in physics laboratory performing experiments. Practical work is expected to bring in behavior changes in the students. The scientific temperament, curiosity, interest and creativity form the bases for this change. Practical work attempts to provide a

body of knowledge through scientific procedures that are demonstrated objectively but today they are often done in a more subjective context.

The laboratory work in physics can be an exciting part of the course or it can be drudgery depending upon the attitude of the students towards it. If they regard it merely as an impediment to getting through the course, probably they will not enjoy it and, furthermore, they will derive very little benefit from it. On the other hand, if their approach towards laboratory work is with the thought that it is an opportunity to learn and with the desire to make the most out of it, then it is almost certain that they will find the time spent on it both profitable and interesting.

Nowadays, unfortunately, it appears as if the students perform experiments just for the sake of marks. This is evident from the fact that they come to the lab unprepared and are completely unaware of the various experiments that they are going to perform or have to perform. They are completely dependent on the teacher for any particular experiment and care very little for the outcomes of the experiment. They do not seem to have the scientific temper or the inclination towards learning anything, at all, in the laboratory which in-turn makes them to be unable to use their time and their resources efficiently.

2. Significance of the study

The significance of the proposed research work is to establish if there are any negative attitudes of the students towards Physics practical work in the laboratory as is evident from their lackadaisical attitude towards lab experiments and their utter lack of enthusiasm and scientific temper; and if so to find out proper ways and means or to suggest remedial measures to help motivate and develop the students' interests in the laboratory. As pointed out earlier, the laboratory is the core component of the study of Physics which helps to prove the various phenomena understood and accepted to be true by the scientists and the scientific community as a whole. In the words of *Richard Feynman – "to guess at the wonderful, simple, but very strange patterns beneath them all, and then to experiment to check again whether we made the right guess"*. It is therefore very important to inculcate a positive attitude towards practical lab work in the minds of the young students who are the budding future scientists of the nation and the whole scientific community.

3. Objective

The objective of this study is to find out the attitudes of the students, of the different colleges of Meghalaya offering BSc at the undergraduate level, towards Physics practical and to assess if there are any negative attitudes, as such, amongst the students towards laboratory experimental works.

4. Experiments Performed By the Students

A representative list of the experiments performed by the BSc students at the undergraduate level as per NEHU's latest syllabi is gives as under:

1. Determination of the co-efficient of linear expansion of a solid by using Pullinger's apparatus and optical lever.

2. Determination of the specific heat of a liquid by the method of cooling.

3. Determination of the co-efficient of thermal conductivity of a good conductor by Searle's method.

4. Determination of the refractive index of a prism by a spectrometer using monochromatic light.

5. Determination of the magnifying power of a telescope by angular method.

6. Determination of the radius of curvature of a lens by Newton's ring method.

7. Determination of the grating constant by using a spectrometer.

8. Determination of the power of the combination of two thin convex lenses in contact by displacement method.

9. Determination of the speed of waves on stretched strings.

10. Determination of the frequency of a tuning fork by Melde's method.

11. Determination of the co-efficient of thermal conductivity of a bad conductor by Lee's method.

12. Determination of the velocity of Ultrasonic waves in liquid.

13. To measure the width of single slit from the study of its Fraunhoffer diffraction.

14. Determination of the wavelength of sodium light using Fresnel's biprism.

15. Determination of the specific rotation of solution using polarimeter.

16. Determination of Young's modulus (Y) of glass by using Cornu's method.

17. Determination of Planck's constant by photocell or by heating method.

18. Determination of specific charge (e/m) of an electron by magnetron/Thomson's method.

19. Determination of the forward and reverse bias characteristics of a Zener diode and to measure the value of breakdown voltage.

20. Determination of the reduction factor of a tangent galvanometer and also the value of horizontal component of earth's magnetic field by electrolysis method.

21. Determination of the monochromatic wavelength by Michelson's interferometer.

5. Sample

For the purpose of the study, 9 (nine) colleges of Meghalaya offering Bachelor of Science (BSc) degree at the undergraduate level were identified and depending upon the enrolment of students in each respective college a sample of about 320 BSc students having Physics as one their subjects were randomly selected, and these were assumed as a complete representative of the entire student community in the state with Physics as one of their subjects. To realize the objective of the study, evidence will be established based on the data collected from the sample surveyed and because of the very nature of the study, these will be reported exclusively through percentages and discussed accordingly.

6. Results Obtained

For the sample surveyed, 39.7 per cent (127)[1] of the students are studying in their 5th Semester, while 59.4 per cent (190) are in their 3rd Semester. This reflects that most of the respondents are the B.Sc. 3rd Semester students. However, it was found that about 0.9 per cent (3) of the sample surveyed did not respond to which

[1] Number in bracket indicates the number of students under study.

class they belong to. These findings are also evident from Figure *(i)* as shown below.

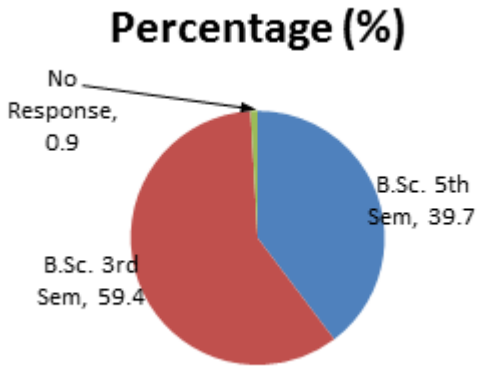


Figure (i): Class-wise Distribution of Students

The distribution of the students surveyed by gender is presented in Figure *(ii)*.

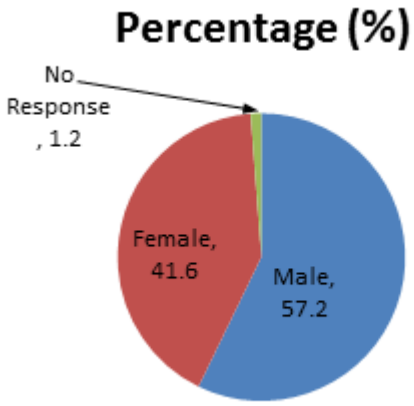
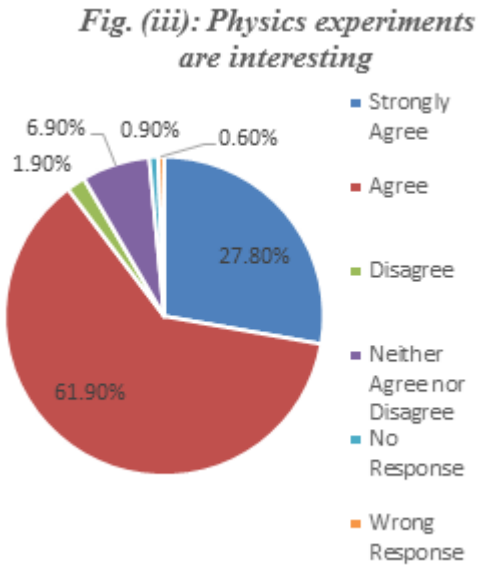


Figure (ii): Distribution of Students by Gender

From the above figure, it is evident that out of all the students surveyed, 41.6 per cent (133) are female while 57.2 per cent (183) are male and 1.2 per cent (4) did not respond to which gender they belong to. While a generalization cannot be made, however, this clearly reflects a gender differential in the number of female students opting for Physics as one of the subjects at their under-graduate level. Male students outnumber female students by around 15 percent.

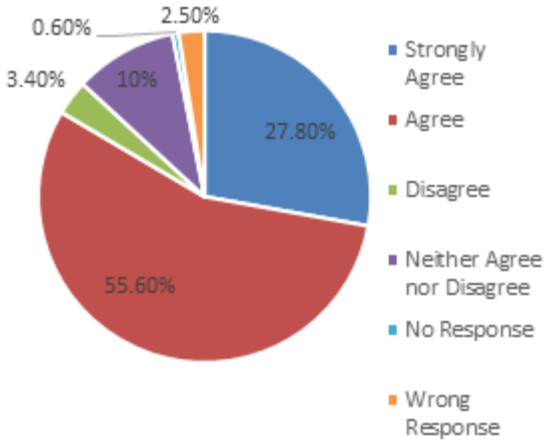
According to the questionnaire, the students had to point out their opinions in four categories – Strongly Agree, Agree, Disagree and Neither Agree nor Disagree; based on a set of questions given to them in-order to find out their attitude towards Physics practical and to assess if there are any negative attitudes, as such, amongst them towards the laboratory experimental works. However, due to certain number of no responses and ineligibility of the responses of the students, two more categories were incorporated for a clear-cut analysis and reporting i.e. No Response and Wrong Response.



The analysis was carried out according to the questions given in the questionnaire and these are reported as discussed below. From the findings, it is evident that more than 80 per cent of the students find Physics experiments as interesting. It is worth mentioning that 9.1 per cent of the surveyed students strongly agree that the

experiments in the BSc Syllabus (NEHU) are still relevant even in the modern context of science, while more than half - 56.6 per cent - of them agree and around 19 per cent were of the opinion that they neither agree nor disagree; which shows that most students are aware of their syllabus and can relate to it even in the modern context of science.

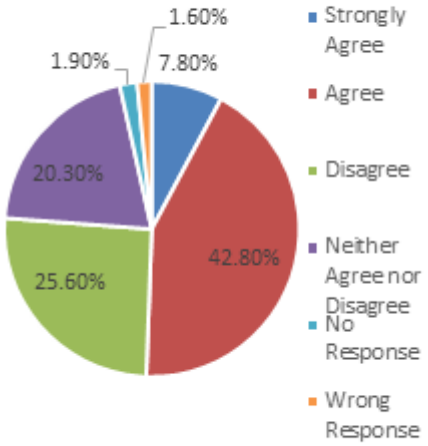
Fig. (iv): Physics experiments will help facilitate research work later



Furthermore, 27.8 per cent responded that they strongly agree and 55.6 per cent agree that the current Physics laboratory work will enable them to apply or facilitate in their research work in the future; Out of all the students surveyed, 30.3 per cent strongly agree, 50.6 per cent agree and only 6.6 per cent disagree that the experiments and laboratory works help them in understanding the theoretical concepts better. Consequently, this encourages the students to like spending time in the laboratory to perform the experiments; which is evident by 12.2 per cent strongly agreeing and exactly half - 50per cent - agreeing to it.

When an individual is inculcated with a concept, this will either encourage or discourage him/her to learn more about it depending upon his interest in the subject concern. If this concept creates an interest or a curiosity in the individual, he/she will try to find out and know more about it. As a result the individual may develop and apply innovative ideas in experimenting ways and means to discover the facts, theoretically as well as practically, where he/she will not merely accept the particular concept on what is being told or taught but will perform and apply them through various experiments. Again, this in-turn will create more interest in that concept or the subject as in case of this study. Also, in the process, the individual enjoy performing these experiments. This is also evident in this study where 7.8 per cent of the students strongly agree and 42.8 per cent agree that they usually question the results of a particular experiment and do not simply accept them.

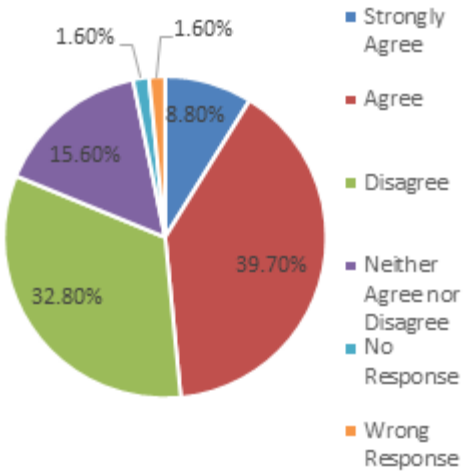
Fig. (v): Questioning the results of a particular experiment



About 4.4 per cent strongly agree and 22.5 per cent agree that they also try to perform the experiments in other ways other than such ways as prescribed in the textbooks. It was seen that some students were also innovative in

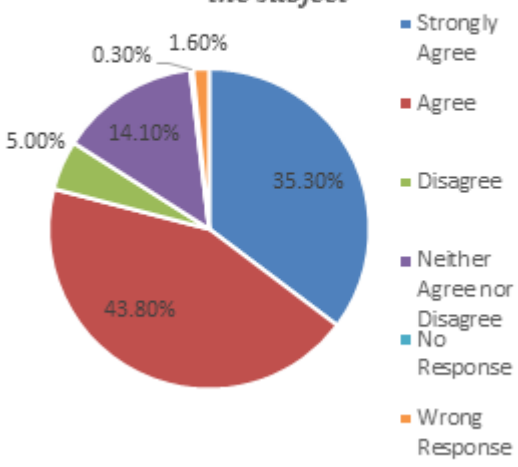
performing their Physics experiments as 8.8 per cent responded strongly agree and 39.7 percent agree that they try to incorporate their own ideas while performing an experiment. However, 32.8 per cent disagree in trying to incorporate their own ideas in performing Physics experiments.

Fig. (vi): Incorporating own ideas while performing an experiment



Also, 35.3 per cent strongly agree and 43.8 per cent agree that performance of experiments in physics increases their interest in the subject.

Fig. (vii): Performing Physics experiments increases interest in the subject



In addition, more than half of the students usually enjoy performing the experiments and do not feel any pressure at all to complete the experiments in trying to keep up with their syllabus.

In today's age, the education scenario in the state appears as if the mindset of most students, as seen by many, is such that they are much more interested in the marks than in understanding the theories or the experiments. However, this is contradictory in this study where only 5.3 per cent of the students strongly agree and 18.8 per cent agree to this saying. However, an enormous 60 per cent of the students surveyed disagree that their interest is in marks only rather than the experiments, while 13.4 per cent were indifferent. Furthermore, 69.7 per cent of the students disagree that they just want to complete the experiments even if they do not understand the experiments whereas only 3.1 per cent strongly agree and 18.1 per cent agree.

Depending upon their attitudes towards the lab work, it may seem like a burden or an opportunity to build up the necessary skills for the future. It was found that 65.3 per cent disagree while only 16.4 percent agree that the lab work should not be a part of their curriculum. Also, 69.1 percent of the students disagree to think that the laboratory work is just an extra burden on their already existing heavy syllabi. The reason for these findings may be that laboratory work not only enables the students to understand more on theoretical aspects of the subject but they also have the opportunity to attain more marks in the subject through performing of experiments in Practical Examination; which is also part of their syllabus. Thus, it is evident that students are fond of laboratory work and about 75 percent of the

students think that laboratory work is not at all tedious and boring but, is in fact interesting and helps to increase their interest in the subject.

7. Conclusion

From the study undertaken, we may conclude that the findings seem to reflect a positive attitude of the students towards Physics practical and, consequently, no negative attitudes towards laboratory experimental works. The results obtained are encouraging as far as the students of Meghalaya are concerned. It is however imperative on the part of the teacher(s) to constantly observe and motivate the students so as to help them overcome any difficulties or doubts, whatsoever, and thus eradicate even an iota of negative attitude from their tender minds before it even start budding. Students should be encouraged to not only understand any particular experiment but also to incorporate their own ideas to any experiment and find out what research works are related to that experiment.

Unfortunately, it appears that in India this is one area of research where researchers have not focused on: to find out the attitudes of students towards lab work which is very crucial for the later stages of their academic pursuits when they will be entering the fields of research. Most of the studies such as the one done by Dr. (Mrs) Urmil Sethi focused on the attitude of students towards the subject of Science in general. From her study she found out there are significant differences in attitudes towards Science between students from rural and urban areas but there are no differences on the basis of gender or socio-economic background. There are other similar studies also which focused mainly on the subject but not on the laboratory work which is a very important aspect and also an integral part of Science learning and teaching.

8. Suggestion for further studies

A more thorough study may be undertaken to find out if there are any differences in the attitudes of boys and girls towards the laboratory work which was not done in this study. Also, one may investigate the reasons behind the low rate of enrolment, in the subject concern, at the research level in the state.

Appendix

List of Colleges covered under the study and their Locations:

1. Shillong College, Laitumkhrah, Shillong, East Khasi Hills District.

2. St. Edmund's College, Laitumkhrah, Shillong, East Khasi Hills District.

3. St. Anthony's College, Laitumkhrah, Shillong, East Khasi Hills District.

4. St. Mary's College, Laitumkhrah, Shillong, East Khasi Hills District.

5. Lady Keane College, Secretariat Hills, Shillong, East Khasi Hills District.

6. Synod College, Jaiaw, Shillong, East Khasi Hills District.

7. Kiang Nangbah Government College, Jowai, West Jaintia Hills District.

8. Tura Government College, Tura, West Garo Hills District.

9. Don Bosco College, Tura, West Garo Hills District.

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Three Famous Physics Experiments with Negative Results and Their Impact

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** Retired. *The article is dedicated with reverence to my respected parents and maan ji.*

Abstract

Three well – known experiments in the history of physics that yielded results contrary to the expected ones, namely, the Michelson – Morley experiment, production of x – rays from fluorescent substances and search for γ – ray resonance fluorescence, together with the far - reaching effect these had on the developments in physics, have been described.

1. Introduction

The beauty of physics lies in the fact that (i) it is self – contained in its logic, and its concepts and laws have evolved in a self – consistent manner from well - founded mathematical reasoning and experimental observations; (ii) the researchers involved in its development use simple as well as cumbersome theoretical techniques based on different branches of mathematics, experimental methods which yield results obtained with modest or sophisticated equipment, and fast computers for simulation as well as analysis of the accumulated data; and (iii) all its findings are theoretically as well as experimentally verifiable, exactly reproducible under similar experimental conditions irrespective of the place of work and are open to complete overhauling if falsified. The experiments are usually performed to unravel the physical aspects of the Nature as per intuition of the physicist, check the validity of a theoretical prediction or to investigate various properties of a system. As such carrying out an experiment requires diligent planning, designing as well as fabrication of apparatus and investment of money depending on the level of sophistication of the instruments and the goal set. Sometimes the outcome of an experiment may be surprisingly at variance with what was anticipated so that the result may be said to be negative in the sense that it contradicts the hypothesis whose verification was sought. In this article, we delineate upon three famous experiments in physics (two of these were carried out in the last two decades of the nineteenth century while the third was performed in the sixth decade of the twentieth century) that belong to this category and, also, bring out the subsequent contributions the outcome of these

experiments made to the growth of physics, in particular, and science in general.

2. Michelson – Morley Experiment

The concept of ether was introduced in science for transmission of mechanical force by Descartes in 1638 and Huygens used this as a medium for propagation of light in the wave theory in 1690. It was put on a sound footing by Fresnel around 1820 when he presented mathematical treatment for different optical phenomena taking light to consist of transverse waves supported by ether. This all - pervading luminiferous medium with self - contradictory mechanical attributes suggested by different workers to enable this to transmit gravitational, electric and magnetic effects besides optical ones became a more acceptable entity after Maxwell's theory of electromagnetic waves and identification of light as one of these in the 1860s. A prominent feature of his equations was that these were independent of the speed of the source emitting the electromagnetic waves. It was followed by the assertion by Lorentz and others that ether must be in a state of absolute rest and that this stationary medium can be used as privileged absolute frame of reference. This, in turn, led to the conclusion that the revolution of earth around the sun and the rotation about its own axis must be movements through this unique medium and an observer on the earth should not only experience an ether wind but must also be able to measure it. However, since the linear speed on the surface of the earth due to rotation is just 1.5 % of the speed in the orbit around the sun, the latter was better option to pursue. This challenging task was taken up by Michelson, who in the eighties of the 19th century, performed many experiments, using his newly invented optical interferometer, to detect and measure the speed of the earth relative to the stationary ether and, thus, to check whether ether existed or not. The initial experiment was done by him alone but it was found to suffer from an error, and the later efforts had Morley as collaborator that has led to the name Michelson – Morley experiment for the outcome of the reasonably precise and laboriously carried out work reported in 1887. Later, they separately performed similar experiments with other colleagues using improved versions of the apparatus.

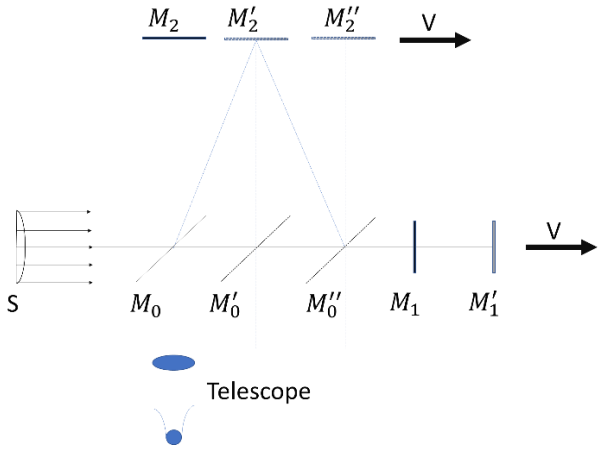

Figure 1. Line sketch of the arrangement for the Michelson – Morley experiment.

A schematic drawing of optical parts of the interferometer used in a typical Michelson – Morley experiment is presented in Figure 1. A parallel beam of monochromatic light from source S is made to fall on a half-silvered mirror or plate M_0 inclined at 45^0 so that the beam is partially reflected and partially transmitted, splitting into two equally intense parts travelling perpendicular to each other along M_0M_1 and M_0M_2. These two beams are reflected from fully coated mirrors M_1 and M_2 mounted on two perpendicular arms, called the longitudinal and transverse arms, respectively, and provided with very accurate micrometer screws for adjustments. On being

reflected at M_1 and M_2, the two beams reach back the beam splitter M_0 and again get partially reflected and transmitted to finally reach the field of view of the telescope kept perpendicular to the longitudinal arm. The superposition of the two beams coming from M_0 leads to interference and the fringes are formed according to the difference in the times taken for traversal along the two arms or the optical paths covered. The effective optical paths $M_0M_1M_0$ as well as $M_0M_2M_0$ are increased manifolds using multiple reflections and usually are adjusted to be equal, say L.

Michelson and Morley had mounted their apparatus on a large stone slab floating in a trough of mercury and the whole assembly was housed in a closed room in the basement of a stone building to eliminate thermal and vibrational effects to maximum possible extent. Their set - up was sensitive enough to measure 0.01 fringe shift for sodium light. The experiment was performed keeping the apparatus such that the motion of the earth was along the longitudinal arm SM_0M_1. Then it was rotated in its own plane through 90^0 without disturbing adjustment to have motion of the earth perpendicular to the new position of SM_0M_1 (Figure 2). The experiment was repeated number of times on the same as well as different days and again after 6 months to rule out any possibility of lack of ether drag in one position of the earth in its orbit. If the earth were at rest with respect to the ether during the first part of the experiment making the effect to be absent, then it would have maximum velocity after 6 months yielding the largest possible effect and vice versa.

We assume that the ether is stationary and the earth as well as the equipment move through this with speed V while the speed of light is c; Figure 1. Alternatively, we may take the earth and the apparatus to be stationary and the ether wind to be blowing with speed V in the opposite direction. In

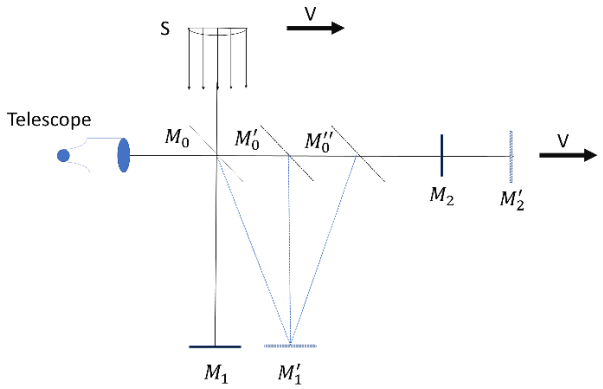

Figure 2. Set up for the Michelson – Morley experiment after being rotated through 90^0.

this way of looking at the things the situation is analogous to the motion of two identical motor boats, having constant speed c' in stationary water, moving in a canal with water current at speed V', such that one boat first goes upstream for distance L and then travels downstream, while the second boat makes a round trip of the same length across the stream. Furthermore, as the treatment pertains to an experiment performed at the end of the 19th century, the Newtonian law of addition of velocities is applied. During its journey from M_0 to M_1 along the longitudinal arm the light traverses distance M_0M_1 = L with relative speed c – V as both the light and the mirrors are moving in the same direction or equivalently the light beam is traveling against the ether current. However, in the return journey from M_1 to M_0, the speed of light relative to the apparatus is c + V. Accordingly, the total time taken by light to cover the path length $M_0M_1M_0$ = 2L is

$$t_1 = \frac{L}{c-V} + \frac{L}{c+V} = \frac{2L}{c\left(1-\frac{V^2}{c^2}\right)}. \qquad (1)$$

The light beam that proceeds along the transverse arm M_0M_2 is obtained by reflection at mirror M_0

which is moving with speed V along the direction of the incident beam. The Huygen's theory as well as the electromagnetic theory [1] show that if a ray of light with angle of incidence i is reflected from a mirror moving along the direction of incidence, the angle of reflection is (i + δ) rather than i (as for a stationary mirror or the one perpendicular to the direction of incidence), where

$$\delta = sin^{-1}\left(\frac{V}{c}\right). \qquad (2)$$

In the present case, since the average speed of earth V in its orbit around the sun is 30 km / s, $\frac{V}{c} \sim 10^{-4}$, we have $\delta \approx sin^{-1}(0.0001) \approx 0.0001\, rad \approx 20.6$", so that the angle of reflection at M_0 is $45^0 20.6$". By the time the reflected beam making an angle δ with the initial transverse line M_0M_2 reaches the mirror M_2, the mirror as well as the light beam are shifted towards right by the same value because both have the same speed V parallel to the longitudinal arm. Therefore, the beam is incident on mirror M_2 at position M_2' such that $\frac{M_2M_2'}{M_0M_2'} = sin\delta$ and angles of incidence and reflection are both δ. The component of speed of light perpendicular to M_0M_1 or along M_0M_2 = L is $(c^2 - V^2)^{\frac{1}{2}}$, that is, the effective speed of light along the transverse arm is reduced from c to $(c^2 - V^2)^{\frac{1}{2}}$. On its back journey towards mirror M_0, the same thing happens and the light beam reaches the beam splitter M_0 at M_0'' such that $sin\delta = \frac{M_0'M_0''}{M_2'M_0''}$. Therefore, the time for the to-and-fro traversal of the path length $M_0M_2'M_0''$ is given by

$$t_2 = \frac{2L}{(c^2 - V^2)^{1/2}} = \frac{2L}{c\left(1 - \frac{V^2}{c^2}\right)^{1/2}}. \qquad (3)$$

A look at Equations (1) and (3) shows that $t_1 > t_2$ making the coverage of the same lengths along the longitudinal and the transverse arms asymmetrical. It may be noted that the multipliers of c in the denominator of these expressions, which are different, reduce to unity for V = 0 and represent the modifications in the value of times t_1 and t_2 due to the motion of earth through ether. The difference between the two traversal times turns out to be

$$\Delta t_{12} = t_1 - t_2 = \frac{2L}{c}\left(\frac{1}{\left(1 - \frac{V^2}{c^2}\right)} - \frac{1}{\left(1 - \frac{V^2}{c^2}\right)^{\frac{1}{2}}}\right)$$
$$\approx \frac{2L}{c}\left[\left(1 + \frac{V^2}{c^2}\right) - \left(1 + \frac{V^2}{2c^2}\right)\right] = \frac{LV^2}{c^3}. \qquad (4)$$

We have used the binomial theorem to write the last but one expression because $\frac{V}{c} \sim 10^{-4}$ and $\frac{V^2}{c^2} \sim 10^{-8} \ll 1$. Clearly, the arrangement compares with great precision the speed of light along the two arms of the interferometer.

The rotation of the set-up through 90^0 interchanges the roles of the two arms making M_0M_2 the longitudinal arm and M_0M_1 the transverse arm; Figure 2. In view of this, the times for traversal of path lengths $M_0M_1'M_0''$ (now transverse arm) and $M_0M_2'M_0$ (the longitudinal arm in the present arrangement) are, respectively,

$$t_1' = \frac{2L}{c\left(1 - \frac{V^2}{c^2}\right)^{1/2}} \qquad (5)$$

and

$$t_2' = \frac{2L}{c\left(1 - \frac{V^2}{c^2}\right)} \qquad (6)$$

so that the time difference becomes

$$\Delta t_{12}' = t_1' - t_2' = -\Delta t_{12}. \qquad (7)$$

Obviously, the total time difference between the two situations of the apparatus is

$$\Delta t = \Delta t_{12} - \Delta t'_{12} = 2\Delta t_{12} = \frac{2LV^2}{c^3}. \qquad (8)$$

The optical path difference corresponding to this time difference is

$$\Delta s = c\Delta t = 2L\left(\frac{V}{c}\right)^2. \qquad (9)$$

Taking the wavelength of the light used as λ, the number of interference fringes shifted by this optical path difference is given by

$$\Delta n = \frac{2L}{\lambda}\left(\frac{V}{c}\right)^2. \qquad (10)$$

Thus, the fringe shift is a second - order effect in V/c.

In their experiment, Michelson and Morley used sodium light for which $\lambda = 5.893 \times 10^{-7}$ m and the effective optical path length L = 11 m so that $\Delta n = 0.373$. Obviously, the expected fringe shift was 37 times the minimum value the interferometer used in the experiment could measure. However, they did not observe any fringe shift. This negative outcome implied that if the earth is moving with respect to the ether then its speed must be less than the relative speed V by a factor of 6 or more because only then one could get $\Delta n < 0.01$, the detectable value for the apparatus. Alternatively, one could think that there was no ether wind (V = 0) or the ether. The possibility of nonexistence of the ether meant that there is no such thing as universal absolute frame of reference and the motion is relative to a particularly chosen frame of reference. Furthermore, $\Delta n \approx 0$ suggests that the speed of light is independent of the motion of the source or the observer as it is same along the two perpendicular arms implying symmetry of traversal of light along these arms. These conclusions, making the idea of ether to be erroneous, were in contradiction with the accepted viewpoint at that time but had to be believed because the experiment was meticulously executed and the results were irrefutable. In fact, Michelson was so disturbed by the outcome of this experiment that he seldom referred to this work for many years.

After Michelson and Morley, this experiment was repeated by Morley and Miller (1902 -1904) and many other workers including Michelson himself, till the third decade of the 20th century, with better precision as well as modified versions of the equipment, which could measure Δn smaller by an order of magnitude. But still the result was null and this was so even when the experiment was performed on a high altitude, or with white light, or the light coming from the sun or a star, or the optical paths were made different along the longitudinal and the transverse arms, or the effect of the motion of earth relative to ether was studied on some electrical phenomena.

Following the invention of Maser and Laser the interest in such investigations got renewed and similar experiments using these were performed in 1955 onwards. The further advancements in the Maser and Laser technologies and development of cryogenic optical resonators has made it possible to detect V / c as small as 10^{-5} or so and the negativity of the result of such experiments leads to the obvious conclusion that light or electromagnetic waves do not need any medium to propagate through, making the age old idea of ether redundant. Besides, the recent experiments have been used to test whether the space is isotropic with respect to the speed of light and the measurements show that the difference between the speed of light along two perpendicular directions is at most of the order of 10^{-17}c.

It is interesting to note that the puzzling null finding of the Michelson – Morley experiment in 1887 implying that a medium was not needed to propagate light waves, prompted many votaries of the ether hypothesis to put forward models which could explain the results of the experiment without discarding the ether. The most prominent and drastic proposal was independently made by Fitzgerald and Lorentz in 1892 who suggested that the length of a moving material body gets shrunk or contracted by a factor $\left(1-\frac{V^2}{c^2}\right)^{1/2}$ along the direction of motion while the dimensions perpendicular to this remain unaffected. This is called the Lorentz – Fitzgerald or simply Lorentz contraction and it leads to reduction of longitudinal arm length from L to $L\left(1-\frac{V^2}{c^2}\right)^{1/2}$. Accordingly, the expression for t_1 gets modified to

$$t_1^L = \frac{2L}{c\left(1-\frac{V^2}{c^2}\right)^{\frac{1}{2}}}, \qquad (11)$$

which is the same as t_2 that remains unchanged, making $\Delta t_{12} = 0$. Similarly, $\Delta t'_{12} = 0$ and hence $\Delta t = 0$, which, in turn, implies that both Δs and Δn are zero. Though this specific purpose hypothesis accounted for no fringe shift in the Michelson – Morley experiment, it lacked a convincing argument in favour of contraction and had to be dropped. This was followed by Poincare's observation in 1899 that absolute motion could not be detected through optical means. Next, in 1904, he stated the principle of relativity to read 'the laws governing physical phenomena must be the same for the observers in the inertial frames of reference having relative translational uniform motion' and used this to show that the Michelson – Morley experiment was intended to achieve something which, in principle, is impossible.

It was at this stage that Einstein (1905), who was not directly influenced by the null result of the Michelson – Morley experiment, argued that the concept of ether is unsound and it should be rejected. Based on the till date observations, he put forward two assumptions and used these to develop a theory for inertial frames in uniform relative motion, which is called the special theory of relativity. Starting from the postulates of (i) the principle of relativity as stated above and (ii) the constancy of speed of light independent of the relative velocity of the source or the observer, Einstein formulated a novel and self – consistent theory that made Maxwell's equations for electricity and magnetism consistent with the laws of mechanics without invoking the concept of luminiferous ether and, thus, making this superfluous; proved that the speed of light in vacuum, namely $c = 3 \times 10^8$ ms^{-1}, provides the upper limit to the speed attainable by a material object; brought in revolutionary changes in the old perception of space and time as separate entities leading to the 4-dimensional interwoven space-time world; led to the Lorentz transformations in a natural way making time, length, the resultant of addition of two velocities, and mass to be variable with velocity (as measured by an observer at rest, faster an object moves more slowly the time passes, less lengthy it is and the greater is its mass, and the changes brought in these quantities become significant when their speeds are a substantial fraction of the speed of light, the so called relativistic speeds); and yielded the illustrious relationship for equivalence of energy and mass, namely, $E = mc^2$. The last result makes energy and mass to be the two facets of the same thing and forms the basis of explanation for the binding energy in the atomic nuclei, the energy

produced in the nuclear reactions, working of nuclear reactors as well as the nuclear bomb, creation and annihilation of electron – positron or proton – antiproton pair, decay of elementary particles, etc. This has been also used to assign mass and momentum to photon and thereby to explain many phenomena. The application of the Lorentz transformations to optical phenomena gives three effects: aberration of light, longitudinal and transverse Doppler shifts, which are widely used for interpretation of various astronomical observations.

The results obtained for the dynamics of moving charged particles in the framework of special theory of relativity have been extensively used in the development of mass spectrometry; particle accelerator technology; different types of electron, proton and ion microscopes; and in the studies pertaining to plasmas and cosmic rays. Though this theory was counterintuitive, yet it had very far reaching philosophical consequences and all its predictions have been found to be experimentally true.

3. Production of X – rays from Fluorescent Materials

In the second half of the 19^{th} century, investigations pertaining to discharge of electricity through gases at low pressure was a topic of great research interest. When a gas at a pressure of about 10^{-3} mm of Hg enclosed in a glass tube fitted with two electrodes (the Hittof-Crookes tube) is subjected to a high potential difference of 10^4 – 10^5 V, a fluorescent glow appears on the walls of the tube behind the anode. This glow was correctly attributed to the cathode rays being emitted by the cathode, which were later identified as electrons. In one of such experiments Roentgen (1895), who had covered his tube with thin opaque black cardboard and placed the apparatus in a reasonably well darkened room, found that a paper screen coated with barium platinocyanide showed fluorescence (which was known to glow on being exposed to the cathode ray beam), even when this was about 2 m away from the tube. After many experiments, he convinced himself that the mysterious penetrating radiation responsible for this fluorescence emerged from the point on the discharge tube where the cathode rays were striking the wall of the tube. He named these as x – rays and thoroughly studied their various basic properties.

The fact that the x-rays appeared to originate from the region of the glass tube where fluorescence occurred, made Becquerel, who had expertise in the study of fluorescence and phosphorescence phenomena, to ponder at whether the reverse process would occur. In other words: will the naturally fluorescent substances exposed to intense beam of light produce the x – rays? With a view to explore this possibility, in February 1896, he started studying double sulfate of uranium and potassium, which was a well - known fluorescent compound. He took a photographic film, carefully wrapped this in a black paper to eliminate the possibility of any effect of the visible and ultraviolet light from the sun rays as well as the fluorescent material, kept some amount of the salt on it and placed the things in the sunlight. He repeated the experiment under varying conditions for few days. On being developed the photographic plate always showed some fogging effect of some radiation making him believe that the fluorescence of the salt by the sunlight produced x-rays. To confirm this finding, he decided to carry out few more experiments. But incidentally the weather became cloudy and he left the salt sample along with the properly wrapped photographic plates in a dark desk

drawer to wait for a sunny day. After few days, out of curiosity, he developed these plates hoping that these would be clear and was, however, surprised to find that these showed strong effect of the radiation though there was no chance for the salt to fluoresce. Further experiments in this direction showed that the blackening of the photographic film was independent of the exposure of the salt to sunlight or of any treatment of the material like being subjected to high temperatures and it occurred even when non-fluorescent uranium compounds were used. This led to negation of the hypothesis that x-rays can be obtained from fluorescence! In fact, Becquerel had discovered a new radiation emanating from uranium salts and the phenomenon was named radioactivity by Marie Curie in 1898.

The discovery of new radiation by Becquerel led to a wide range of fruitful researches for many years to come and laid the foundation of nuclear physics. On being subjected to magnetic and electric fields it turned out that the radiations are composed of positively charged, negatively charged and electrically neutral rays. These were, respectively, called alpha, beta and gamma rays and later thorough investigations into the properties of these radiations led to their identification as doubly ionized helium atoms, electrons, and photons, respectively. This prompted scientists to call the two charged components alpha and beta particles as well. Soon after Becquerel's work, M Curie and P Curie started working on this problem and isolated two more radioactive elements from the uranium ore pitchblende in 1898 itself and named these polonium and radium. This was followed by the discovery of radioactivity in thorium, radon and numerous other elements.

The outcome of experiments on scattering of the alpha particles from the radioactive source Polonium by the gold and other metal foils led to the idea of nuclear atom model which later became the basis of quantum theory of atomic structure. It was found that the radioactivity is associated with random decay of atoms or nuclei having characteristic disintegration probability and the fraction of the nuclei left at time t is given by the Rutherford – Soddy decay formula

$$\frac{N(t)}{N(0)} = e^{-\lambda t} = e^{\frac{-0.693t}{T_{1/2}}}. \qquad (12)$$

Here, λ is probability per unit time for decay of each nucleus in the sample and $T_{1/2} = 0.693/\lambda$ is the time in which the number of radioactive nuclei is reduced to half of the initial number at t = 0. The parameters λ and $T_{1/2}$ are, respectively, called decay or disintegration constant and half-life. Furthermore, the decay rate R(t) = - dN(t)/dt, known as the activity of the material, is independent of the chemical changes and the ambient physical conditions of the substance implying that it is a nuclear process rather than an atomic one. It may be noted that R(t) = N(t) λ so that R(t) / R(0) is also governed by Eq. (12). A particularly important result of the studies pertaining to radioactive transformations was the discovery of atoms having the same atomic number and the chemical properties but different atomic weights (the isotopes) which, in turn, explained non-integral atomic weights of some elements. The radioactive nucleus is usually referred to as radionuclide.

The observation that the radiations emanating from some radioactive materials were highly energetic made scientists believe that these may be able to induce transmutations in other elements and this was found to be so when in 1919 Rutherford and his coworkers succeeded in transforming nitrogen into oxygen by bombarding

this with energetic alpha particles. This experiment also led to the discovery of proton. It was followed by the discovery of induced nuclear transformation in various elements brought about by bombardment with different particles in 1930 onwards and the discovery of neutron in 1932 by Chadwick. These works paved way for theory of composition of the nucleus and further experiments on radioactive materials provided an enormous amount of information about different nuclei. In 1934, I Curie and F Joliot found that products of some induced nuclear transmutations were themselves radioactive leading to the beginning of artificial radioactivity. The disintegration in the artificially obtained nuclei is also governed by the same laws as in the natural radionuclides.

As it stands now the term radioactivity is used for a process by which the nucleus of an unstable atom loses energy by spontaneously emitting radiation such as alpha particles, beta particles, gamma rays, positrons, conversion electrons, etc and, in general, transformed into another nuclear atom. The disintegrating nucleus is called the parent nucleus while the product so obtained is known as the daughter nucleus.

Besides the academic consequences of the study of radioisotopes, these have had a considerable practical importance. These are used for diagnosis and treatment of diseases; to sterilize disposable medical supplies as well as food items; for treatment of seeds to improve crop yield; to monitor various industrial as well as mining processes and to nondestructively check the quality of the products; for generation of electric power in nuclear plants as well as space crafts; for radiometric dating of rocks, minerals and fossils; for studying biological, chemical and ecological processes using radionuclide tracers; and so on. It may be mentioned that the radiation from the radionuclides ionize the material through which they pass and this aspect is harmful for the living tissue. A tissue exposed to such a radiation can repair itself if the damage is small otherwise it can be quite hazardous as this can cause cancer, leukemia and alterations in the DNA leading to genetic deformations.

4. Observing γ – ray resonance fluorescence

The phenomenon of resonance in oscillating mechanical systems, acoustic instruments and devices, and electric circuits was well understood by the end of the 19^{th} century. As a natural extension of this, Rayleigh put forth the idea of observing resonance fluorescence in atomic systems in 1894. It was realized by the research works reported by Wood in 1904 and 1912 and by Dunoyer in 1912. This, in turn, helped in strengthening the Bohr's quantum theory of atom presented in 1913. In fact, when a parallel beam of yellow light from a sodium lamp, which has mean wavelength 589.3 nm corresponding to photon energy 2.10 eV, is incident on a flask or a tube of glass containing sodium vapours at low pressure, the sodium atoms in the ground state absorb the photons and undergo transition to the excited state. This state being short lived, the excited atoms return to the ground state emitting the characteristic yellow light of the same wavelength as incident, in all the directions. The intensity of light in the original direction of incidence gets reasonably reduced. Since this process involves absorption and emission of light having the same frequency or wavelength it is resonant fluorescence scattering which has been thoroughly investigated since then and explained in the framework of quantum mechanics and quantum electrodynamics.

The study of γ – ray photons emitted by radionuclides during the beginning of the 20^{th} century showed that these are emitted when a nucleus goes from an unstable higher energy state

to a lower energy state or the ground state. This similarity to the emission of light from the atoms in the excited state led to speculation that γ – ray resonance fluorescence must be observable in the nuclei. First experimental effort in this direction was made by Kuhn in 1929. However, success eluded the physicists for nearly two decades though different workers made many consecrated attempts. This phenomenon was observed by Moon in 1950 and by some other researchers in the coming years, but their experimental arrangements were reasonably complicated.

To understand the reason for this situation, we note that when an isolated stationary atom or nucleus of mass m in the excited state of energy E_e undergoes transition to ground or normal state of energy E_g, it emits a photon of energy $E_\nu = E_e - E_g$, which has linear momentum E_ν / c along the direction of propagation. The conservation of linear momentum demands that the emitting entity should suffer a backward kick or recoil of translational kinetic energy

$$E_R = (E_\nu / c)^2 / 2m. \qquad (13)$$

Since the light photons produced during atomic transitions have energy $1 - 10$ eV and the γ – rays emitted in nuclear transitions carry energy few keV to few MeV, the recoil energy for the latter in the case of the same atom will be $10^6 - 10^{12}$ times that for the former. For example, the energy of recoil experienced by the sodium atom on emission of yellow light will be 10^{-10} eV, while its value for the isotope with mass number 24, which emits γ photon of energy 472.2 keV, is 4.9 eV. Obviously, the energy lost in the recoil is taken from the energy E_ν of the photon leaving energy $E_\nu - E_R$ for this to carry with. Furthermore, when a photon is absorbed, the absorbing atom or nucleus gets a forward kick to conserve linear momentum and this necessitates the availability of energy $E_\nu + E_R$ for its transition from the ground state to the excited state.

It may be mentioned that according to Heisenberg's indeterminacy principle an energy state with mean life-time τ is uncertain by $\Gamma_n = \hbar/\tau$, leading to a spread in the energy distribution about the peak value where Γ_n is taken to be full width at half maximum (FWHM) of the energy profile which is plot of intensity versus frequency or wavelength. Γ_n is called natural line-width. Here, $\hbar = 1.055 \times 10^{-34}$Js is reduced Planck's constant. Clearly, Γ_n is nonzero for an excited state which is short lived while it is zero in the case of ground state for which τ is infinitely large. Accordingly, the radiation emitted during transition from energy state of life-time τ to the ground state is not a precise spectral line at E_ν but has a spread or width. In addition, collision among atoms in the gas and the temperature dependent motion of the emitting as well as the absorbing entities further enhance this line-width to, say, Γ. The latter broadening arising due to thermal agitation is referred to as Doppler or thermal broadening and is proportional to square root of product of recoil energy and absolute temperature of the system. Both these broadenings even change the shape of the energy profile. Thus, the energies of the emitted and the absorbed photons have a distribution of FWHM Γ_e and Γ_a, respectively, around the energy values $E_\nu - E_R$ and $E_\nu + E_R$, as shown in Figure 3. It is important to note that the spread or width of the profile is greater for smaller intensities.

It is clear from Figure 3 that resonance fluorescence will take place only if there is significant overlap of energy profiles of the emission and absorption radiations shown as the region between ABC and the energy axis in the figure. This can happen if the separation $2E_R$ between the two peaks is small, which is so in the case of light. The relatively large magnitude of $2E_R$ for the γ – rays renders the observation quite difficult and the early remedial steps used to compensate it complicated the things. Since the natural line-width is a characteristic of the nucleus for a specific γ – energy, the possibility of overlap of the emission and absorption energy profiles could be increased by broadening the line-widths

by increasing the temperature of both the source and the absorber gases.

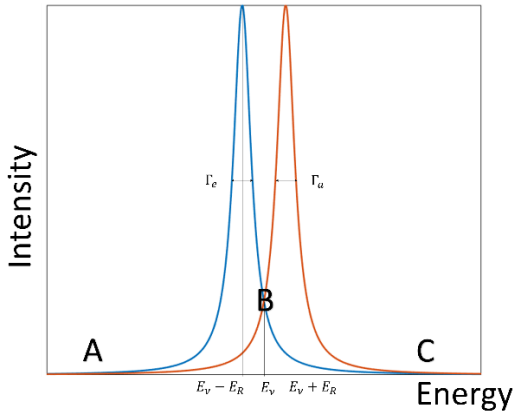

Figure 3. Profiles for the emission and absorption lines. Resonance fluorescence of γ – rays will occur only if the two profiles overlap considerably.

In 1957, Mössbauer was investigating nuclear resonance scattering of 129.4 keV γ – rays emitted by the excited nuclear energy state of ^{191}Ir having mean life-time $\tau = 1.85 \times 10^{-10}$ s and natural line-width $\Gamma_n = 3.56 \times 10^{-6}$ eV. The recoil energy E_R for this isotope is 4.7×10^{-2} eV while the Doppler broadening at temperature of 300 K is about 6.9×10^{-2} eV, which is 10^4 times the value of Γ_n and is the dominating contribution to the effective line-width Γ. Since $\Gamma > E_R$ the profiles for the emission and absorption energies would show some overlap for small intensities making it possible to observe γ – ray resonance fluorescence at 300 K. As a part of his studies, Mössbauer cooled the emitter as well as the absorber to decrease the Doppler broadening (this will have value nearly 4.0×10^{-2} eV at 100 K) and thereby reduce the overlap of the emission and the absorption energy profiles making resonance fluorescence less probable or even absent. However, in contrast with his expectations, the resonance fluorescence was found to increase significantly making the outcome of the experiment to be negative.

This observation was marvelously explained using Lamb's theory for neutron capture by atoms in a crystal, according to which kinetic energy associated with the recoil of an atom in a solid may either produce quanta of vibrational energy of the crystal lattice (known as phonons) or cause the entire crystal to recoil due to strong binding which is nearly 10^3 times the recoil energy of an isolated atom. In the latter case, since mass of the crystal is very large as compared to the atom (even in a fine powder a crystallite contains 10^{15} or so atoms which increases m in Equation (13) by that factor) the recoil energy E_R of the system is extremely small part of the energy of transition and probability of occurrence of this situation is finite. The almost zero value of E_R also renders the Doppler broadening negligible. Also, the collisions amongst the atoms are absent. Accordingly, for a fraction of emitted and absorbed γ – rays the recoil energy as well as the Doppler broadening (which can be further decreased by carrying out experiment at low temperatures) are so small that the peaks in the crystals of the same material will be very close making the profiles only due to natural broadening overlap each other resulting in resonance fluorescence. Later, Mössbauer confirmed the existence of such peaks through differently planned experiment. Thus, in a single experiment using the emitting and absorbing nuclei in crystals he eliminated the effect of recoil due to momentum associated with the γ - photons, got rid of the Doppler broadening and obtained resonance fluorescence for γ – rays with natural line-width. This resonance arising from recoil-free emission and absorption of γ – rays without producing any phonons in the two lattices has been named Mössbauer effect, recoilless γ – ray resonance or zero-phonon γ – ray resonance. The phenomenon was soon developed to a new precise spectroscopic technique bearing the discoverer's name. Obviously, the Mössbauer effect will be intense if the fractions of emitted and absorbed

recoil free γ - photons, known as the Mössbauer fraction, for the source and absorber materials are large.

It must be emphasized that spectra pertaining to the Mössbauer effect have extraordinarily small line-width close to the natural broadening for the γ – ray involved (there can be some contributions due to electronics of the experiment and other arrangements of the set-up making the line-width somewhat larger) so that these are sensitive to very minute variations in the nuclear transition energies. Thus, it provided a technique to investigate any changes in the relevant nuclear energy levels as small as the effective line-width. In the case of most popular 14.4 keV γ – ray from isotope ^{57}Fe, $\tau = 1.41 \times 10^{-7}$ s and $\Gamma_n = 4.67 \times 10^{-9}$ eV, so that a difference $\sim 10^{-9}$ eV or so can be observed in a material containing this nucleus. Since this change can be measured in γ – ray energy $E_\nu = 1.44 \times 10^4$ eV, the fractional measurable change is $\sim 10^{-13}$, that is, 1 part in 10^{13}. Of course, this precision, known as the sharpness of resonance, differs from one isotope to another. Nonetheless, this feature has been immensely exploited in manifold applications of Mössbauer spectroscopy.

It is important to note that the change in the energy levels of a nucleus manifests itself as shift and splitting (due to partial or complete removal of the degeneracy of nuclear levels) leading to corresponding changes in the transition energy E_ν. These, in turn, result in shift in the position of the γ – ray resonance line and production of many lines as per the selection rules governing the transition from the excited state to the ground state. This modification, known as hyperfine structure, can occur because of internal electric and magnetic fields acting on the nuclei of atoms in solids produced by the surrounding electrons and ions, and, also, under the influence of applied high magnetic field and pressure. The interaction between the nucleus and its surroundings that brings about this change is referred to as hyperfine interaction and, generally, its characteristic energies are more than the width of the γ – ray lines. The hyperfine structure is experimentally observed by giving forward and backward Doppler velocity to the emitter or absorber crystal over a wide range of few cms^{-1} to take care of possible values of shift and splitting because a speed v produces Doppler energy shift

$$\Delta E = (v/c)\, E_\nu. \qquad (14)$$

The nature of the hyperfine structure is determined by the characteristics of the excited and the ground states of the nucleus and by the strength of bonding and arrangement of the atoms in the solids containing the emitting and absorbing nuclei. Electric-monopole, electric-quadrupole, and magnetic-dipole interactions are the main types of hyperfine interactions of an atomic nucleus with its surroundings. An electric-monopole interaction arises from the Coulombic interaction of nuclear charge with the electrostatic field generated at its location by the surrounding electrons. This interaction leads to a line shift in the absorption spectrum if the emitter and the absorber are chemically different or if the distribution of the electric charge in the nucleus in the ground state and in the excited state are different from each other. This shift, called isomeric or chemical shift, gives information about the electron density at the nuclear site and hence about the oxidation state, spin state, and chemical bonding of atoms in the solid. Since the electrons in the s orbits have maximum probability of overlap with the nuclear region these contribute the most to the isomeric shift. Furthermore, the information about the charge distribution in atomic nuclei can also be obtained from study of chemical shifts. The interaction of the nuclear electric quadrupole moment with the electric field gradient in the surrounding (the electric-quadrupole interaction) produces splitting of the nuclear levels resulting in the quadrupole splitting. The findings about this are used to get knowledge about the extent of departure from the cubic symmetry of the charges around the nucleus in a solid, the electron configurations of atoms and ions and the quadrupole moments of atomic

nuclei. Magnetic - dipole hyperfine interaction has its origin in the interaction between the nuclear magnetic dipole moment and the magnetic field acting at the nuclear site. It is usually observed in the magnetically ordered (ferromagnetic, antiferromagnetic, and ferrimagnetic) materials in which strong magnetic fields ~ 10^6 oersteds are present. Such fields can also be obtained by placing the sample in external magnetic fields of same order of magnitude. The magnetic-dipole interaction completely removes the degeneracy of the nuclear states and, thus, splits both the ground and the excited states producing several lines in the absorption spectrum. Investigation of magnetic hyperfine structure can be used to determine magnetic dipole moment of the nucleus and the magnetization of the crystal lattice. The temperature variation of the latter provides a tool to precisely determine the temperature at which the magnetic ordering vanishes, that is, the Curie and Neel temperatures. Besides, Mössbauer spectroscopy has been very useful in studying various aspects of the lattice dynamics of the solids. It may be mentioned that application of high pressures ~ 10^5 – 10^6 bars significantly affects the electron distribution in the atoms and, also, the structure of the solids which influence the lattice dynamics as well as the hyperfine structure and hence can be investigated with this technique.

In view of the above, the Mössbauer spectroscopy has been widely used as a powerful technique in physical, chemical, biological and earth sciences and in archaeology, mineralogy (on earth, moon and mars) and different industries. One of the earliest applications of this effect was to observe in the laboratory the red shift of gamma radiation in the gravitational field of the earth and check the validity of Einstein's principle of equivalence. In 1960, Pound and Rebka carried out a systematic study of the effect of gravitation on 14.4 keV γ – ray photons emitted by ^{57}Fe (frequency $\nu = E_\nu/h$ = 3.48 x 10^{18} Hz) subjected to a fall through a height of 22.56 m and found the fractional change in frequency to be (1.05 ± 0.10) times the value predicted by Einstein's general theory of relativity, and known as fractional Einstein shift. This result was significantly improved to (0.9990 ± 0.0076) in 1965 using an improved version of the set-up so that the theory was verified to a very high degree of accuracy. It is worth pointing out that since resonance absorption occurs only for a single nuclear species it can be used when the atoms in whose nuclei the Mössbauer effect is being observed are present in a solid in the form of impurities. This feature has been exploited to study the electronic states of impurity atoms in metals and semiconductors and the characteristics of the vibrations of impurity atoms in crystals. In addition, this technique has been very useful in investigating various aspects, particularly magnetic properties, of different types of nanomaterials.

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Problem

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Problem No1

Two pith balls of negligible radii, carrying unlike charges q_1 and q_2 respectively, are suspended in equilibrium by insulated threads of negligible masses and of lengths $10\sqrt{3}\,cm$ and 10 cm, respectively, inclined at angles 60^0 and 30^0 to the horizontal. If the points of suspension of the threads are $25\sqrt{3}$ cm are horizontally apart, find the weights of the pith balls and tensions of the threads.

Solution to problem No 1:

Let W_1 and W_2 be the weights of the pith balls A and B and T_1 and T_2 the tensions of the threads AX and BY, while the line joining the centres of the pith balls is of length d and makes an angle θ to the horizontal.

The pith ball A is in equilibrium under the action of three forces:

(i) Weight of the sphere A acting vertically downwards
(ii) Tension T_1 acting along the thread AX
(iii) Force F of attraction between unlike charges of magnitudes q_1 and q_2

along AB. Hence by Coulomb's law

$$F=\frac{q_1 q_2}{d^2} \qquad (1)$$

Permissivity of the medium is assumed to be 1.

Similarly the pith ball B is in equilibrium under the action of three forces:

(i) Weight of the sphere B acting vertically downwards
(ii) Tension T_2 acting along the thread BY
(iii) The same force of attraction given by (1) but in the opposite direction

This is illustrated in figure 1.

Considering the equilibrium of the pith ball A , whereas by geometry $\angle T_1, W_1 = 150^0$, $\angle W_1, F = 90^0 + \theta$, $\angle F, T_1 = 120^0 - \theta$ (2)

By Lami's theorem

$\frac{W_1}{sin\angle F,T_1} = \frac{T_1}{sin\angle W_1,F} = \frac{F}{sin\angle W_1,T_1}$

Or, $\frac{W_1}{sin(120^0-\theta)} = \frac{T_1}{sin(90^0+\theta)} = \frac{F}{sin150^0}$ (3)

$W_1 = F(\sqrt{3}\cos\theta + \sin\theta)$ (4)

$T_1 = 2F\cos\theta$ (5)

Similarly for equilibrium of the pith ball B whereas by geometry,

$\angle T_2, W_2 = 120^0, \angle W_2, F = 90^0 - \theta, \angle F, T_2 = 150^0 + \theta$

By Lami's theorem one gets

$\frac{W_2}{sin\angle F,T_2} = \frac{T_2}{sin\angle W_2,F} = \frac{F}{sin\angle W_2,T_2}$ (6)

$\frac{W_2}{sin(150^0+\theta)} = \frac{T_2}{sin(90^0-\theta)} = \frac{F}{sin120^0}$ (7)

$W_2 = F(\cos\theta - \sqrt{3}\sin\theta)/\sqrt{3}$ (8)

$T_2 = 2F\cos\theta/\sqrt{3}$ (9)

By geometry pertaining to figure 1 we have

$10\sqrt{3}\cos60^0 + d\cos\theta + 10\cos30^0 = 25\sqrt{3}$

$d\cos\theta = 15\sqrt{3}$ (10)

$10\sqrt{3}\sin60^0 - 10\sin30^0 = d\sin\theta$

Or $d\sin\theta = 10$ (11)

Equations (10) and (11) imply

$d = 5\sqrt{31}$ (12)

so that

$\sin\theta = \frac{2}{\sqrt{31}}$ and $\cos\theta = \frac{3\sqrt{3}}{\sqrt{31}}$ (13)

Using (1) and (12)/(13) in (4),(5),(8) and (9) we find the weights of the pith balls

and tensions of the threads:

$W_1 = \frac{q_1q_2}{(5\sqrt{31})^2}\left(\frac{9}{\sqrt{31}} + \frac{2}{\sqrt{31}}\right) = \frac{11q_1q_2}{775\sqrt{31}}$ (14)

$W_2 = \frac{q_1q_2}{(5\sqrt{31})^2}\left(\frac{3}{\sqrt{31}} - \frac{2}{\sqrt{31}}\right) = \frac{q_1q_2}{775\sqrt{31}}$ (15)

$T_1 = \frac{\sqrt{3}q_1q_2}{775\sqrt{31}}$ (16)

$T_2 = \frac{6q_1q_2}{775\sqrt{31}}$ (17)

Considering the whole system consisting of two charged pith balls and taut threads, mutual force of attraction between the pith balls will not come into play and hence resolving all the forces horizontally and vertically for equilibrium,

$T_1\cos60^0 = T_2\cos30^0$ (18)

$W_1 + W_2 = T_1\sin60^0 + T_2\sin30^0$ (19)

It is worthwhile to note that the magnitudes of the weights and tensions given by

Equations (14) to (17) satisfy equations (18) and (19).

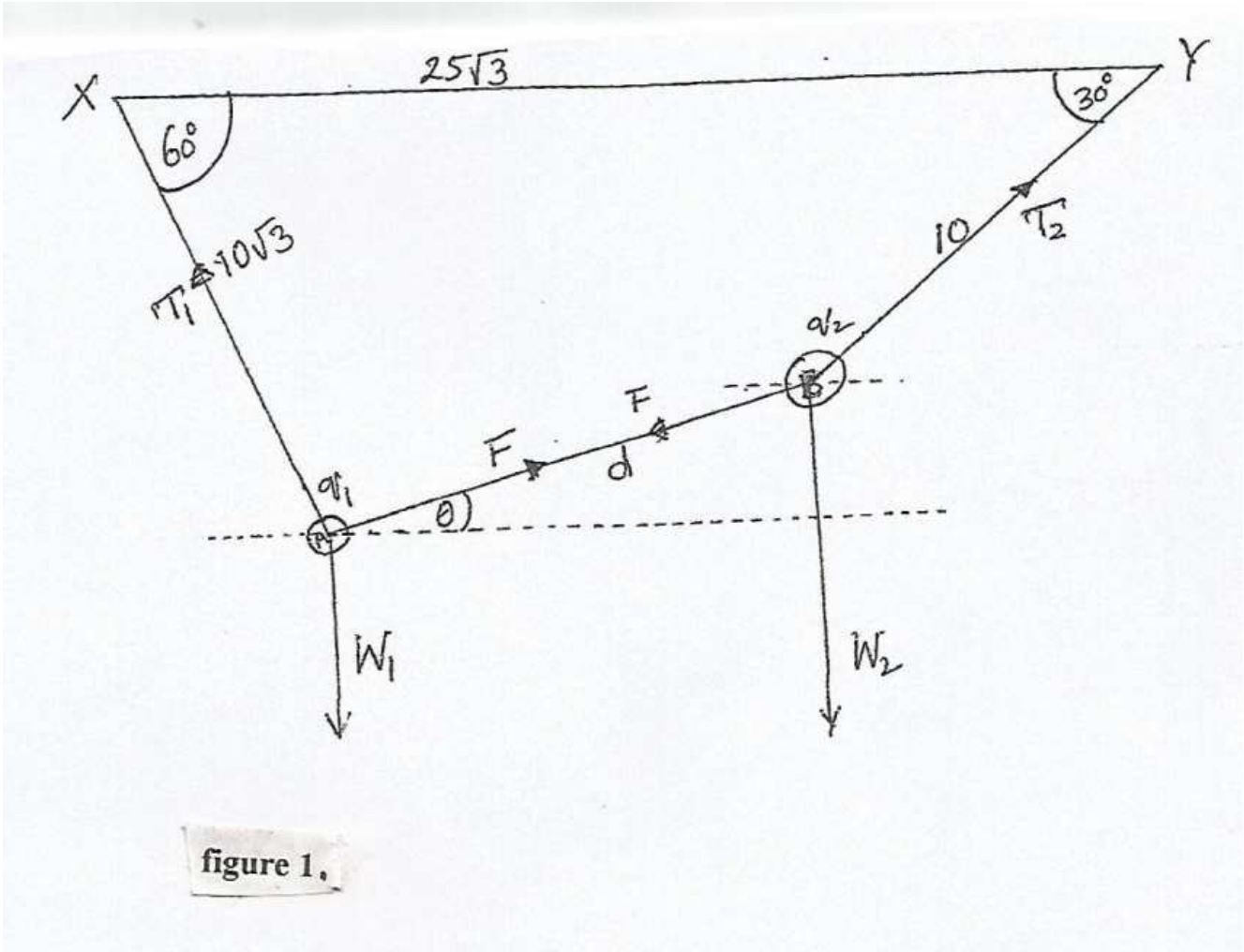

figure 1.

Errata

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There are two mistakes in the article 05 published in Volume 34, Issue 1.

1. Page 3, line 8 at the left hand. "an" should be "and".
2. Page 6, line 2 from bottom at the right side. "$(m+M)d\overrightarrow{r}_c$" should be "$(m+M)\overrightarrow{g}\cdot d\overrightarrow{r}_c$".